New Approximations for Temporal Vertex Cover on Always Star Temporal Graphs

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Algorithmic Aspects of Temporal Graphs IX
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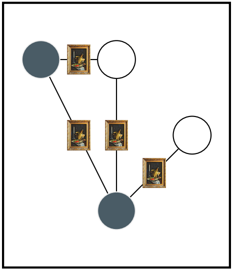
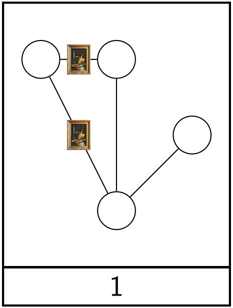
Minimum (Static) Vertex Cover

Given an undirected graph $G = (V, E)$ and a parameter $k \in \mathbb{N}$, what is the smallest k and a subset $V' \subseteq V$ such that $|V'| = k \land \forall \{u, v\} \in E : u \in V' \lor v \in V'$?

For example:



Temporal Graphs



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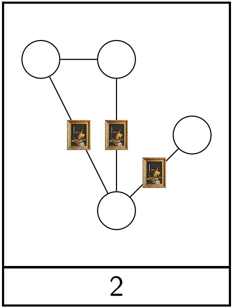

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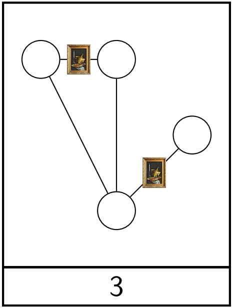

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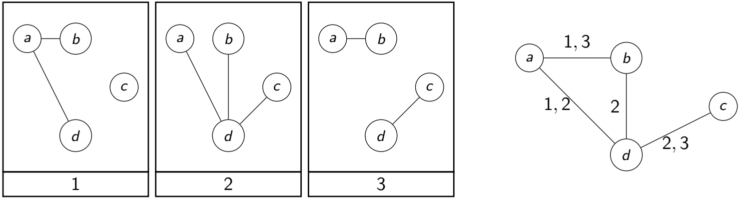


A temporal graph is a pair (G, λ), where $G = (V, E)$ is an underlying (static) graph and $\lambda : E \to 2^{\mathbb{N}}$ is a time-labeling function which assigns to every edge of G a set of discrete-time labels.

Lifetime T - highest assigned label

Temporal Graphs and Vertex Cover

How do we cover edges of temporal graphs?

Temporal Graphs and Vertex Cover

How do we cover edges of temporal graphs?

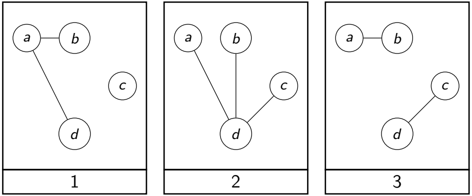
An edge e is considered temporally covered by a vertex appearance (w, t), when

1. w is an endpoint of e
2. e is active at t, e.i. $t \in \lambda(e)$

Hence, (w, t) covers all adjacent edges to w, which are active at t.

Temporal Vertex Cover (TVC)

A temporal vertex cover of (G, λ) is a temporal vertex subset S of (G, λ) such that every edge $e \in E(G)$ is temporally covered by at least one vertex appearance $(w, t) \in S$.



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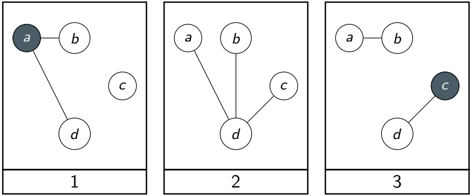

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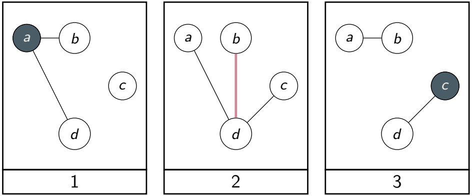

NOT a valid TVC

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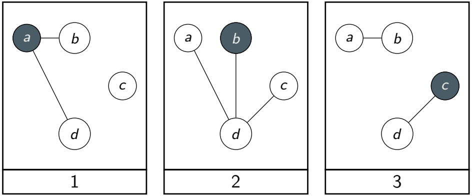

Valid TVC

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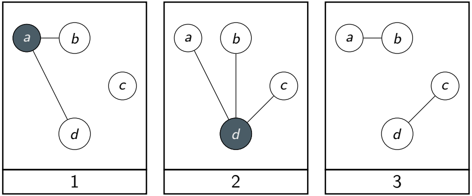
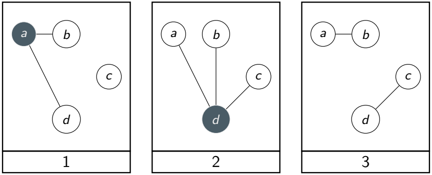


Minimum TVC

Sliding Window Temporal Vertex Cover (TVC)

A sliding Δ-window temporal vertex cover of (G, λ) is a temporal vertex subset S of (G, λ) such that for every time window W_t and for every appearing edge $e \in E[W_t]$, e is temporally covered by a vertex appearance $(w, t) \in S$ in W_t.



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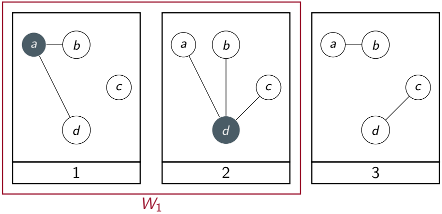

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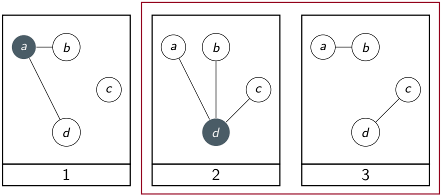

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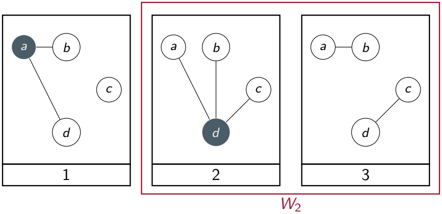

NOT a valid $\Delta = 2$ SW-TVC

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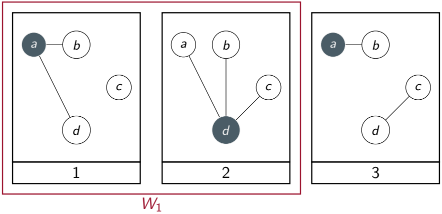

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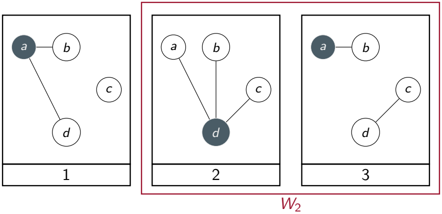

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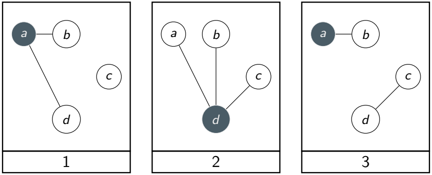
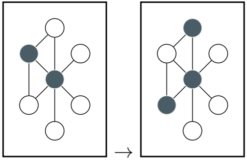


Valid $\Delta = 2$ SW-TVC

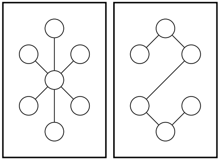
Both TVC and SW-TVC

NP-hard

How can these be solved?



Approximation



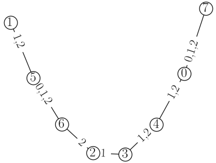
Restriction

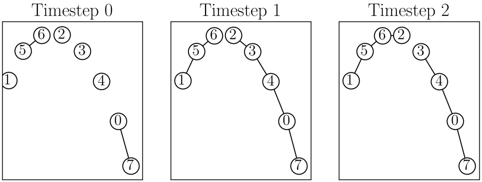
Temporal Graph Classes

Underlying vs Always

Underlying vs Always

For Example:
Underlying path (or just *path*)

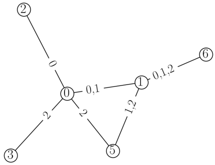


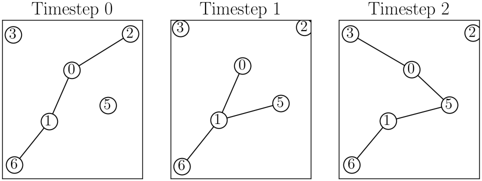


The underlying structure is a path!

Underlying vs Always

For Example:
Always degree at most d

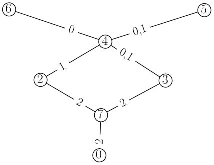


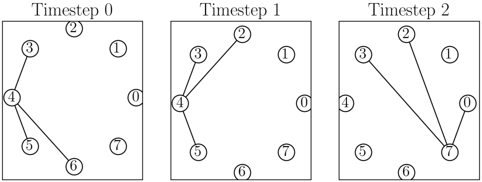


Every snapshot has at most degree d!

Underlying vs Always

For Example:
Always star temporal graphs





Every snapshot is a star!

Related Work

Class	**NP-hard**	**Algorithms**	**Approx.**	**Time**
Arbitrary	✓	exact [Akr+20]	1	$\mathcal{O}(T\Delta(n+m)\cdot 2^{n(\Delta+1)})$
		EXACT [Ham+22]	1	$\mathcal{O}(T\Delta^{\mathcal{O}(m)})$
Path/cycle	x	[Ham+22]	1	$\mathcal{O}(Tn)$
Always path/cycle	x	[Ham+22]	1	$\mathcal{O}(Tn)$
Always degree at most d	✓	*D-APPROX* [Akr+20]	d	$\mathcal{O}(Tm)$
		D-1-APPROX [Ham+22]	$d-1$	$\mathcal{O}(T^2m^2)$
Always star	✓			

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		D-1-APPROX [Ham+22]	$d-1$	$\mathcal{O}(T^2m^2)$
Always star	✓	*STAR-SC*	$2\Delta-1$	$\mathcal{O}(T)$
		STAR-ACOV	$\Delta-1$	$\mathcal{O}(Tm\Delta^2)$

SW-TVC for Always Star Temporal Graphs

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Trivial idea: *STAR-SC*

$\rightarrow$ Include the star center of every time step with an active edge

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Theorem

STAR-SC approximates SW-TVC on always star temporal graphs with $T \geq \Delta$ *and* $\Delta \geq 2$ *with ratio* $2\Delta - 1$ *in* $\mathcal{O}(T)$ *time.*

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SW-TVC for Always Star Temporal Graphs

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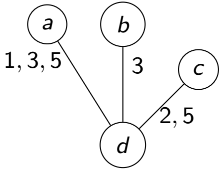
Algorithm: *STAR-ACOV*

SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

Example with $\Delta = 3$

	1	2	3
	av	av	av
$\{d, a\}$	1	0	1
$\{d, b\}$	0	0	1
$\{d, c\}$	0	1	0



SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

Example with $\Delta = 3$

	$\downarrow$ 1	2	3
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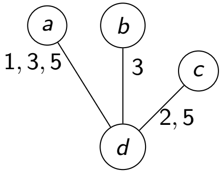

SW-TVC: {(d,3)}

SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

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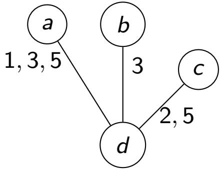


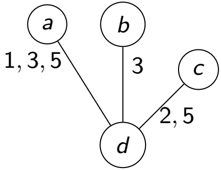
SW-TVC: {(d,3),(c,2)}

SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

Example with $\Delta = 3$

	4	↓ 2	3
	av	in	in
$\{d, a\}$	0	0	1
$\{d, b\}$	0	0	1
$\{d, c\}$	0	1	0



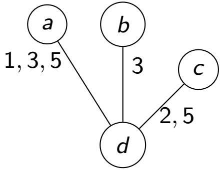
SW-TVC: {(d,3),(c,2)}

SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

Example with $\Delta = 3$

	4	5	3
	ex	av	in
$\{d, a\}$	0	1	1
$\{d, b\}$	0	0	1
$\{d, c\}$	0	1	0



SW-TVC: {(d,3),(c,2)}

SW-TVC for Always Star Temporal Graphs

Always Star Approximation Algorithm: *STAR-ACOV*

Example with $\Delta = 3$

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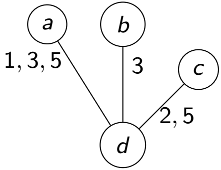

SW-TVC: {(d,3),(c,2),(d,5)}

SW-TVC for Always Star Temporal Graphs

Theorem

STAR-ACOV approximates SW-TVC on always star temporal graphs with $T \geq \Delta$ and $\Delta \geq 2$ with ratio $\Delta - 1$ in $\mathcal{O}(Tm\Delta^2)$ time.

SW-TVC for Always Star Temporal Graphs

Theorem

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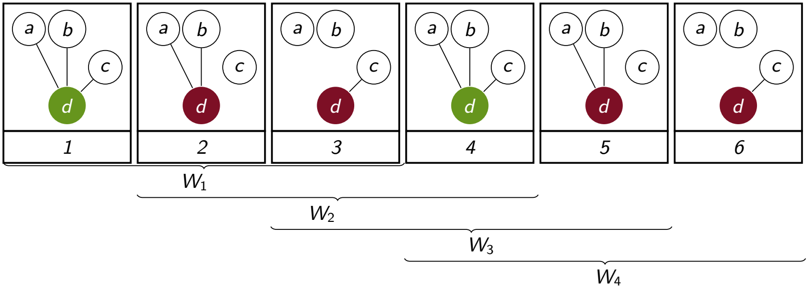
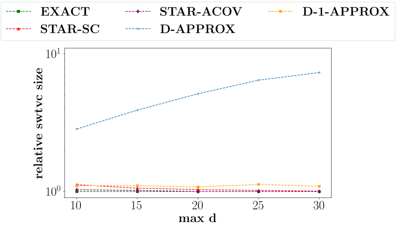
Proof idea

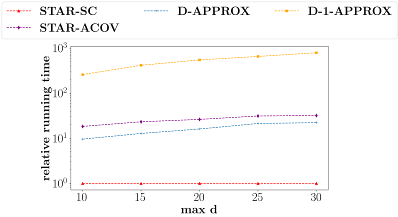


Experimental Evaluation

On Always Star Temporal Graphs:



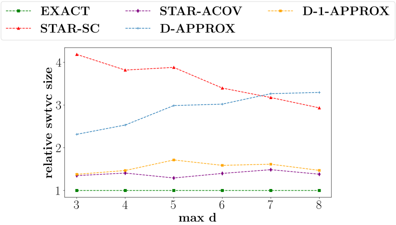
(a) Approximation ratios of 3- and 4-TVC on always star temporal graphs



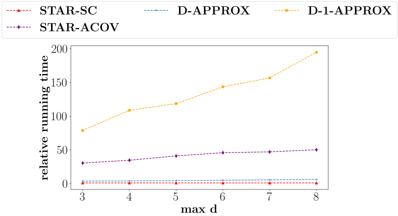
(b) Runtime comparison on always star temporal graphs

Experimental Evaluation

On Always Star Temporal Graphs:
Edge case: $\Delta > d$



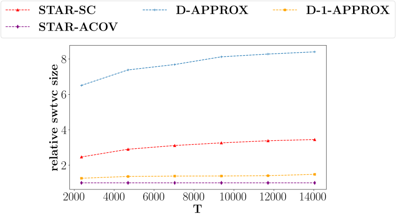
(a) Approximation ratios of 20-TVC on always star temporal graphs



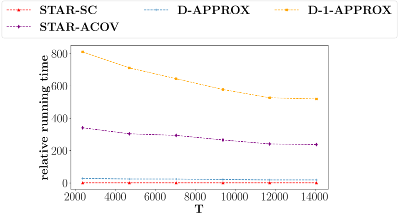
(b) Runtime comparison on always star temporal graphs

Experimental Evaluation

On Always Star Temporal Graphs (Larger):



(a) Approximation ratios of 16-TVC on always star temporal graphs

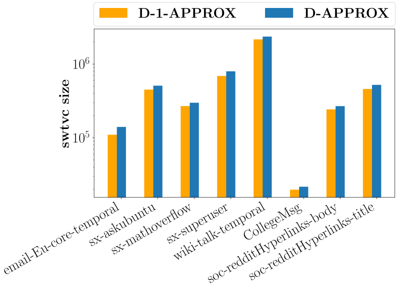


(b) Runtime comparison on always star temporal graphs

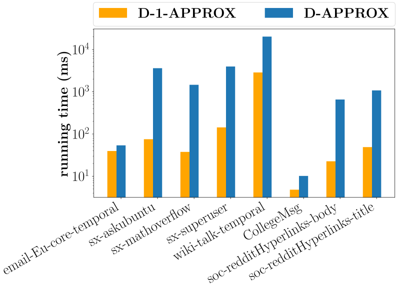
STAR-ACOV algorithm compared to *D-1-APPROX* algorithm:
Solution size improvement: 40.46%
Runtime improvement: 218.69%

Experimental Evaluation

On Real Life Instances (from SNAP):



(a) Solution size of 64-TVC



(b) Runtime

D-1-APPROX algorithm compared to *D-APPROX* algorithm:
Solution size improvement: 11.58%
Runtime improvement: 858.35%

Detour: *D-APPROX* [Akr+20]

Idea: A single edge graphs can be solved linearly in $\mathcal{O}(T)$

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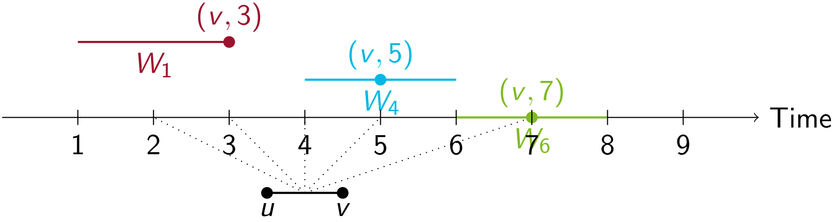
Idea: A single edge graphs can be solved linearly in $\mathcal{O}(T)$

In every window,

choose a latest possible vertex appearance to cover it.



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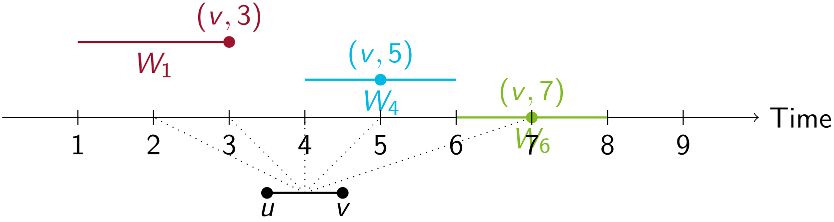


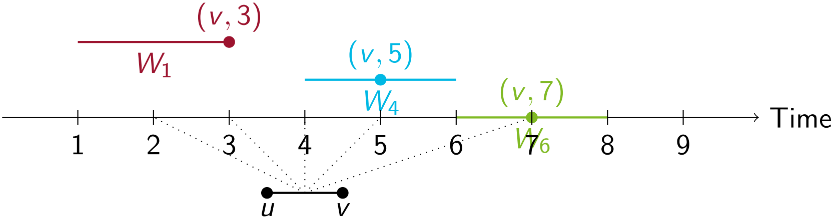
Repeat this for every edge and combine the solutions

Detour: *D-APPROX* [Akr+20]

Idea: A single edge graphs can be solved linearly in $\mathcal{O}(T)$

In every window, where the edge appears,

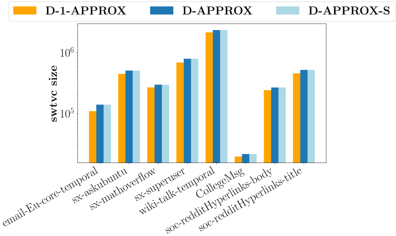
choose a latest possible vertex appearance to cover it.



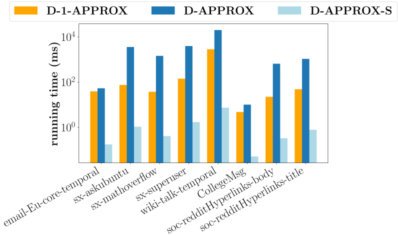
Repeat this for every edge and combine the solutions

Experimental Evaluation

On Real Life Instances:



(a) Solution size of 64-TVC



(b) Runtime

D-APPROX-S algorithm compared to *D-APPROX* algorithm:
Solution size improvement: 0%
Runtime improvement: 259 288%

Conclusion

This Work:

- First experimental results for (SW-)TVC Algorithms
- Practical optimizations
- New Approximations for (SW-)TVC on Always Star Temporal Graphs

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Future Work:

- Consider restrictions to other temporal graph classes
- Collect real life always star temporal graphs and experiment with them

Other Works

Beyond temporal graphs, my work spans:

- **Dynamic graph algorithms**
  Maintaining solutions under edge/node updates
- **Parallel graph algorithms**
  Scalable implementations for large-scale graph processing
- **Bridging theory and practice**
  Turning theoretically efficient algorithms into usable systems

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[Ham+22] Thekla Hamm, Nina Klobas, George B. Mertzios, and Paul G. Spirakis. "The Complexity of Temporal Vertex Cover in Small-Degree Graphs". In: *Thirty-Sixth AAAI Conference on Artificial Intelligence, AAAI 2022, Thirty-Fourth Conference on Innovative Applications of Artificial Intelligence, IAAI 2022, The Twelveth Symposium on Educational Advances in Artificial Intelligence, EAAI 2022 Virtual Event, February 22 - March 1, 2022*. AAAI Press, 2022, pp. 10193–10201. URL: https://ojs.aaai.org/index.php/AAAI/article/view/21259 (visited on 07/23/2023) (cit. on pp. 27, 28).