Data Structures for Temporal Queries on Dynamic Temporal Graphs*

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Algorithmic Aspects of Temporal Graphs IX
Royal Holloway, University of London
United Kingdom
July 6th, 2026

* Based on a joint work with Luciano Gualà, Stefano Leucci, Guido Proietti, and Alessandro Straziota

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- Design a compact **data structure** (a.k.a. **oracle**) that
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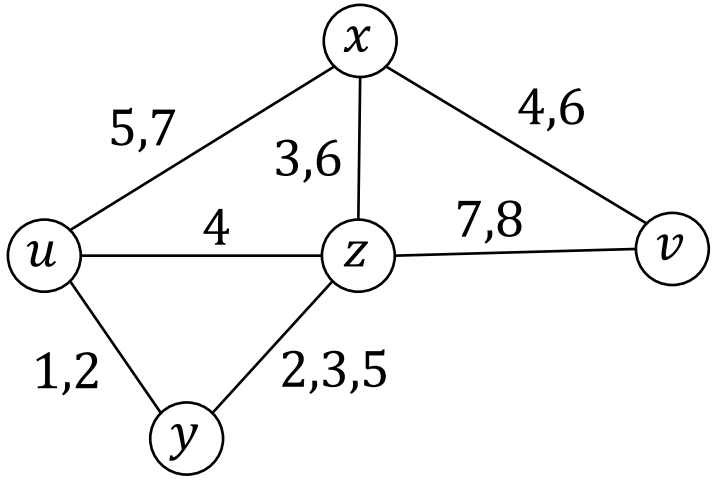
Design oracles with good tradeoffs among
size, query time, and update time (only if dynamic)

Temporal graph model



Temporal graph

- undirected graph
- each edge e has a set $\lambda(e)$ of integer time labels

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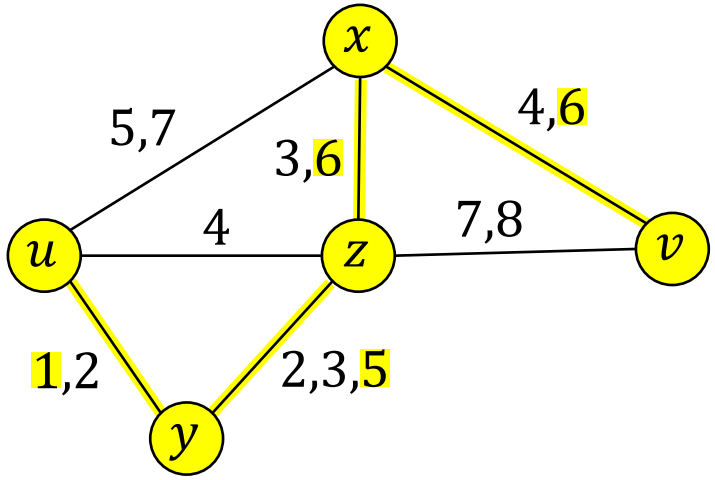

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A $u - v$ path P, with a selected $t_e \in \lambda(e)$ for each edge e in P, s.t. the sequence of traversed t_e's is monotonically non-decreasing

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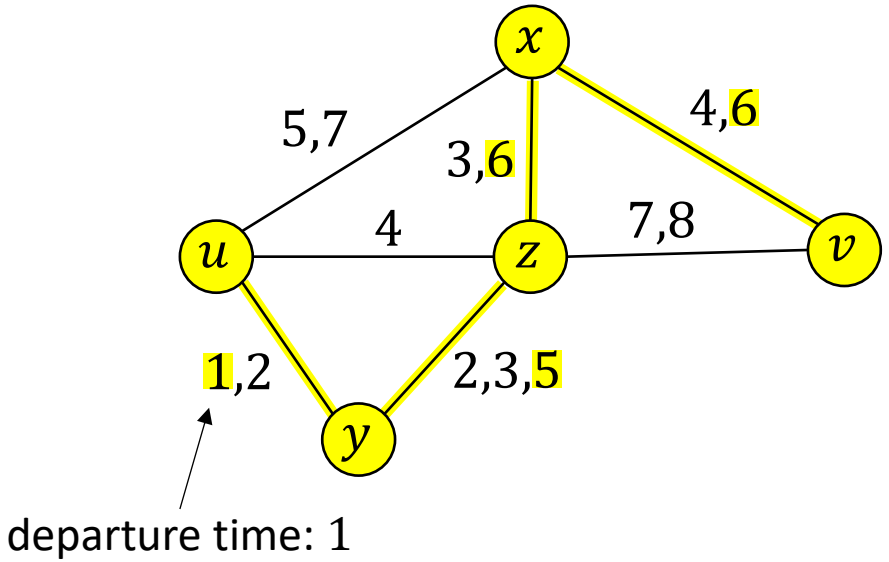

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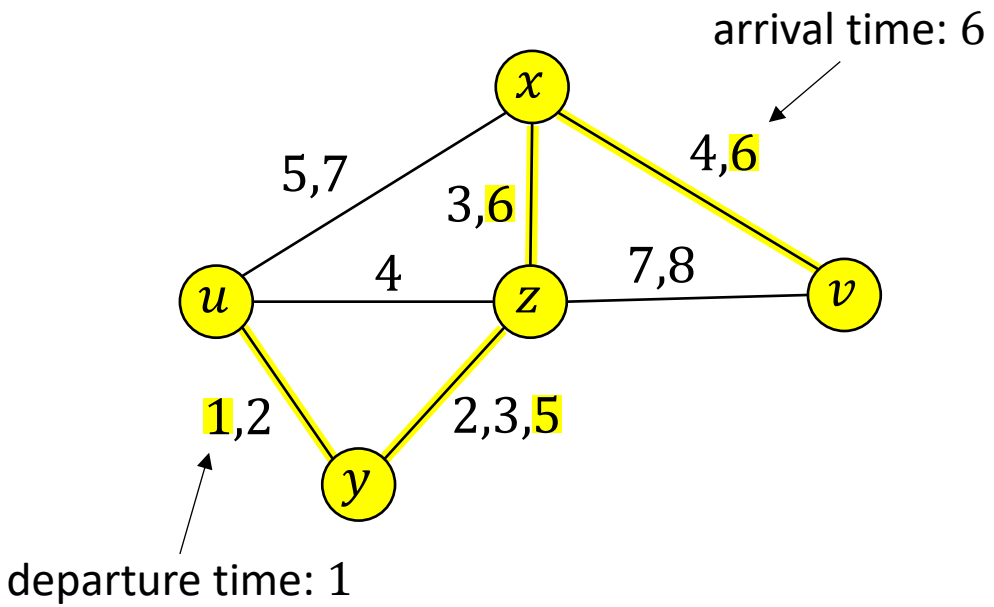

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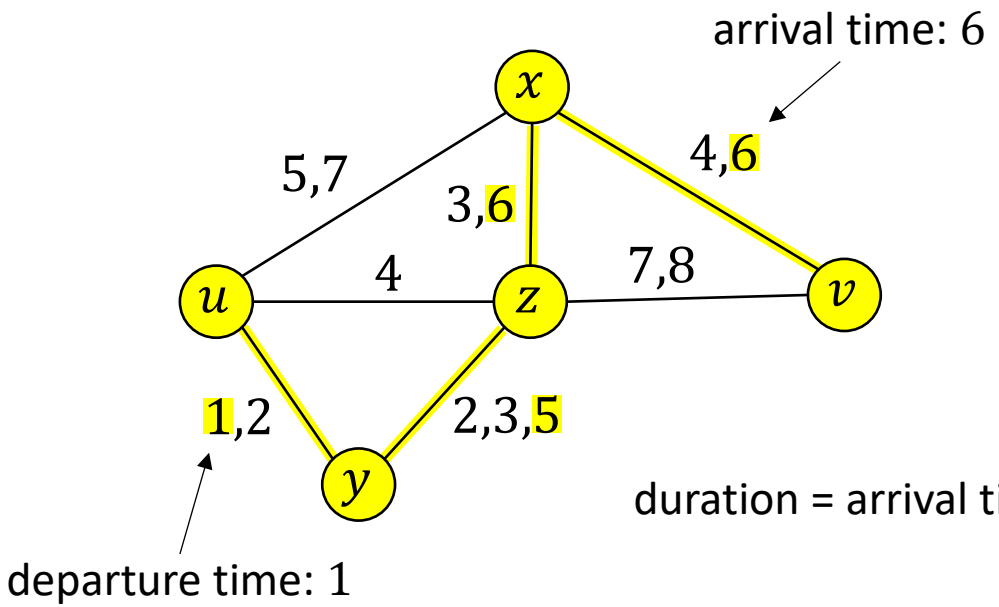

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duration = arrival time – departure time = $6 - 1 = 5$

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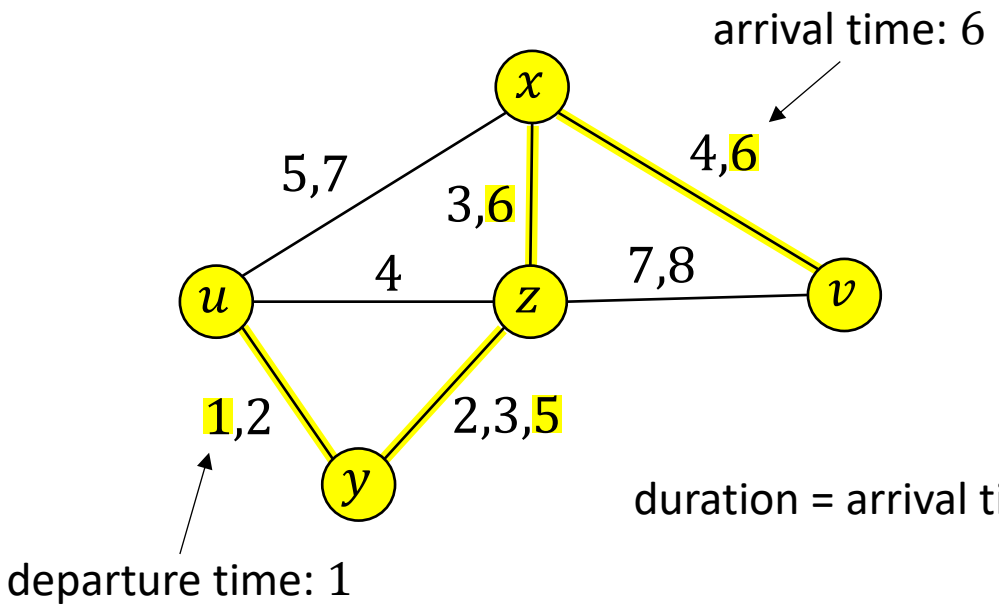

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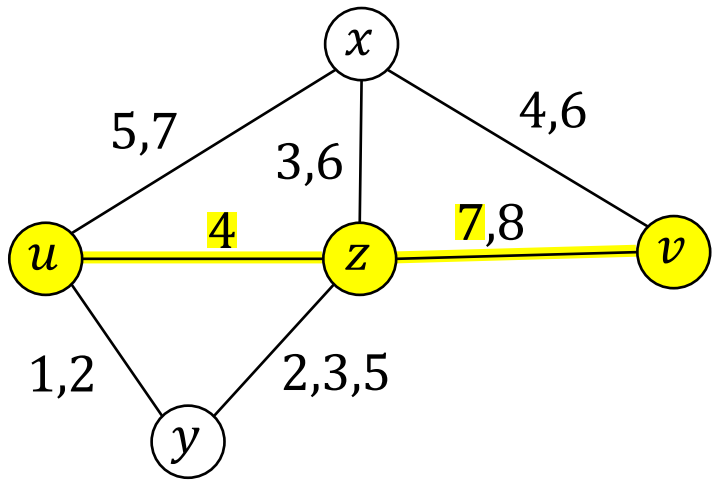
All the results we will see can be adapted to strict temporal paths

Queries

Temporal Reachability $TR(u, v, [d, a])$

Existence of a temporal $u - v$ path restricted to the time interval $[d, a]$



$TR(u, v, [3,7]) = TRUE$

Queries

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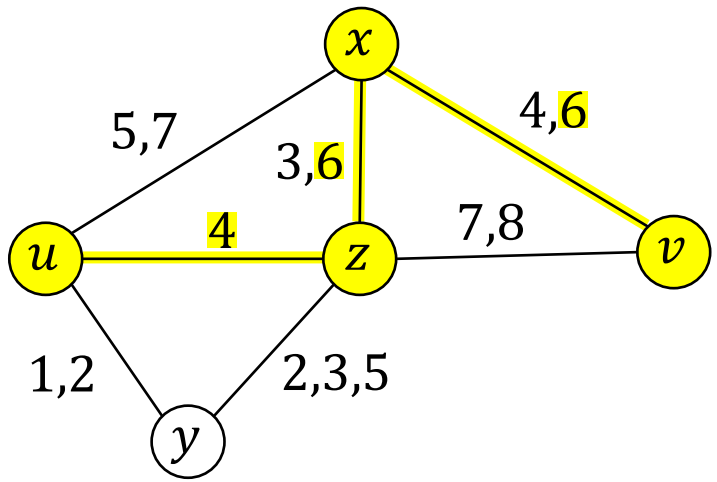
Earliest Arrival $EA(u, v, d)$

Minimum arrival time of a temporal $u - v$ path with departure time $\geq d$



$EA(u, v, 3) = 6$

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Earliest Arrival $EA(u, v, d)$

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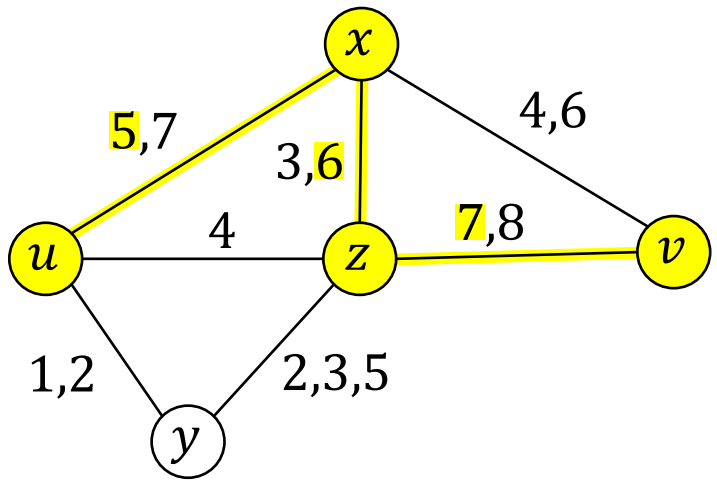
Latest Departure $LD(u, v, a)$

Maximum departure time of a temporal $u - v$ path with arrival time $\leq a$



$LD(u, v, 7) = 5$

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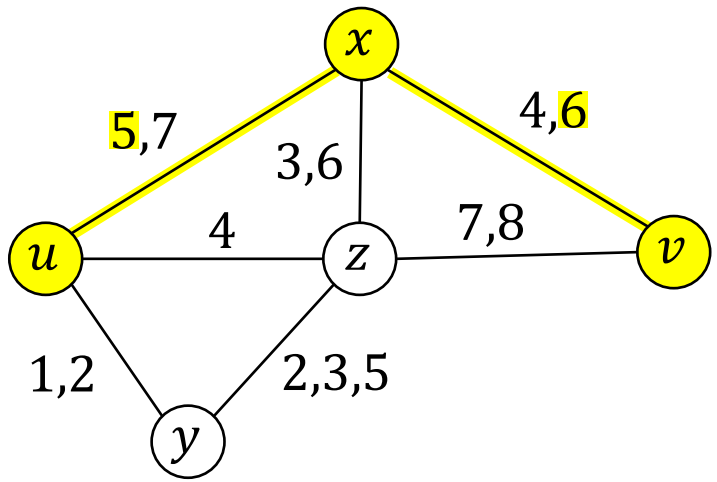
Min Duration $MD(u, v, [d, a])$

Minimum duration of a temporal $u - v$ path restricted to the time interval $[d, a]$



$MD(u, v, [3,7]) = 1$

Useful observations

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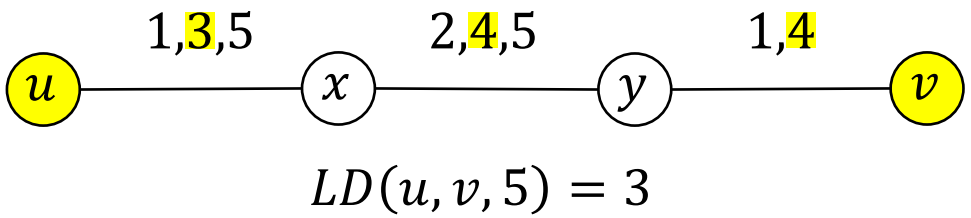
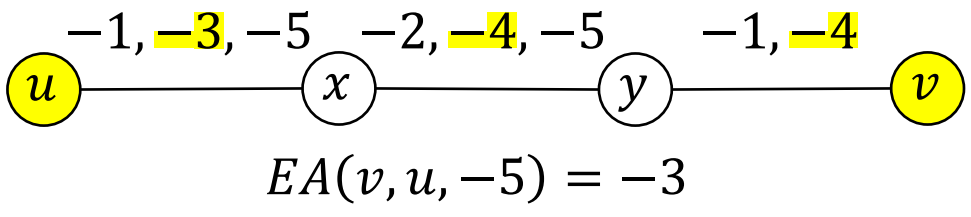


$LD(u, v, 5) = 3$



$EA(v, u, -5) = -3$

State of the art prior to our work

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- **Brito, Albertini, Travençolo, and Navarro, The Computer Journal 2024**: compact representation in practice

Our contribution

[1] B., Gualà, Leucci, Proietti, and Straziota, ALGOWIN 2025

[2] B., Gualà, Leucci, Proietti, and Straziota, ISAAC 2024

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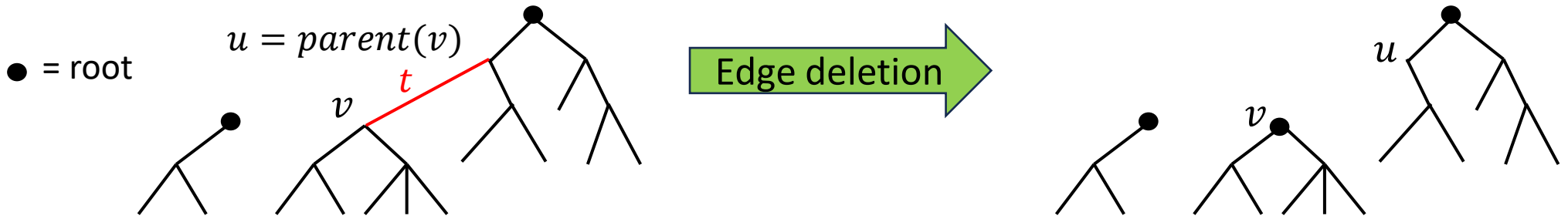

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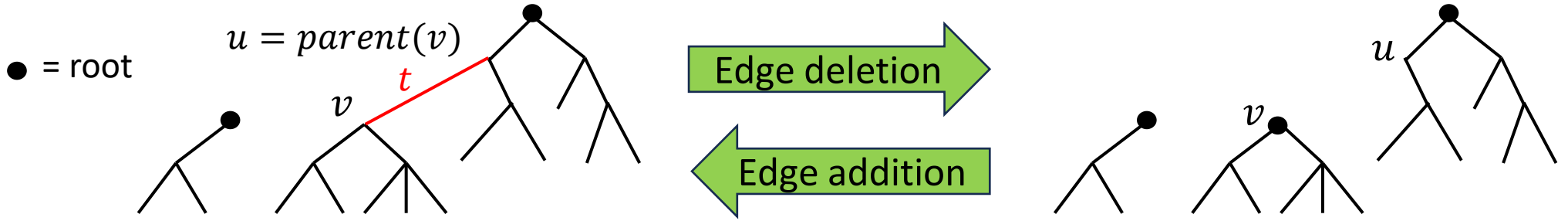

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- Tradeoffs in [1] are asymptotically tight, up to polylog factors, under the STRONG SET DISJOINTNESS CONJECTURE
- Edge updates in a forest of rooted trees
  - Edge deletion: delete an edge $(parent(v), v)$ having only one time label (v is the root of the new formed tree)
  - Edge addition: add edge (u, v) with time label t (v must be the root of the absorbed tree)
- If we add latencies to forests of rooted trees
  - Uniform latencies: same trade-offs
  - Non-uniform latencies
    - Incremental and decremental scenarios (i.e., only insertions or deletions)
    - same trade-offs, but update time bounds become amortized

Results presented in this talk

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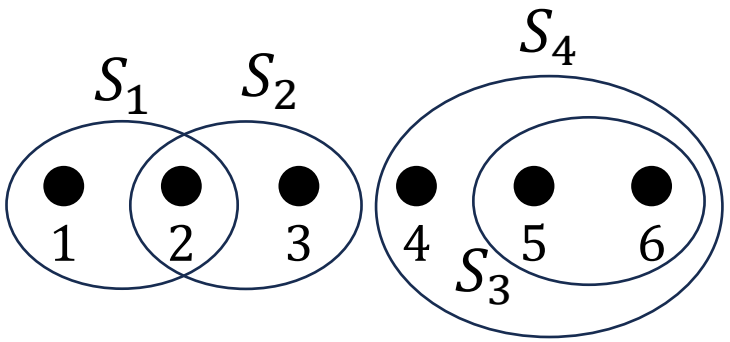
The conditional LB of oracles supporting MD queries

SET DISJOINTNESS PROBLEM: Given k subsets $S_1, \dots, S_k$ of a universe U with $N = \sum_i |S_i|$, design an oracle that can answer to queries *«Given i and j, are S_i and S_j disjoint?»*.



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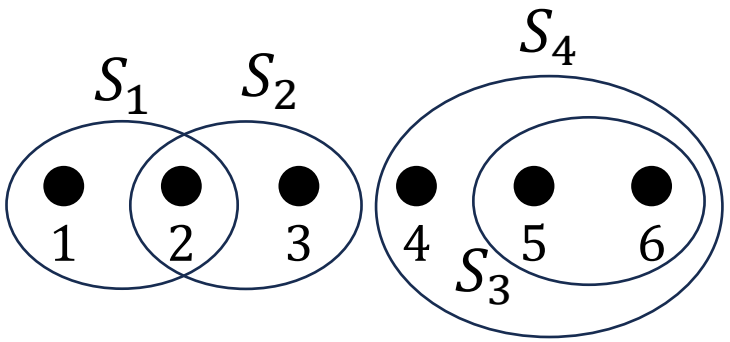

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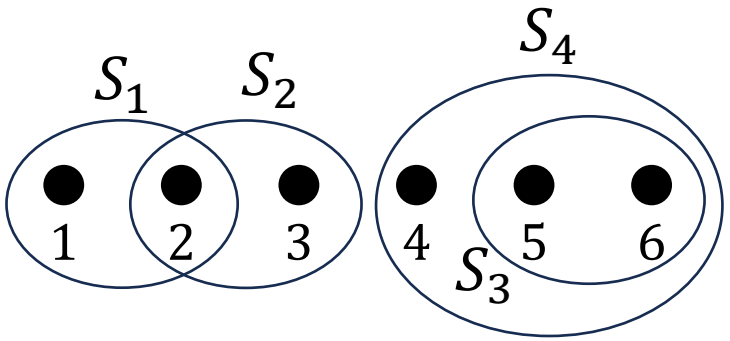

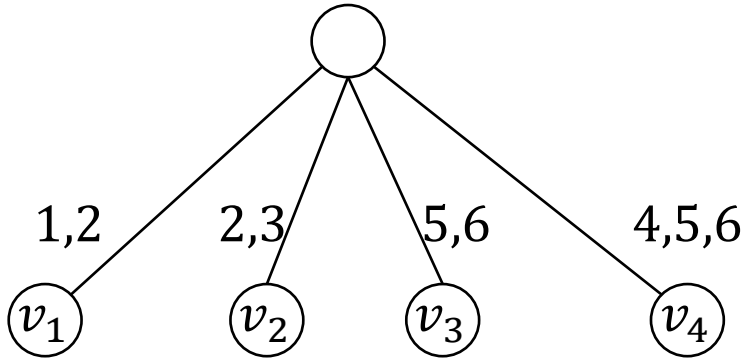

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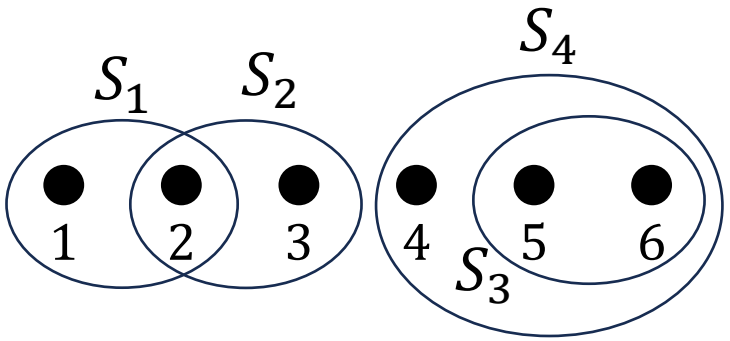

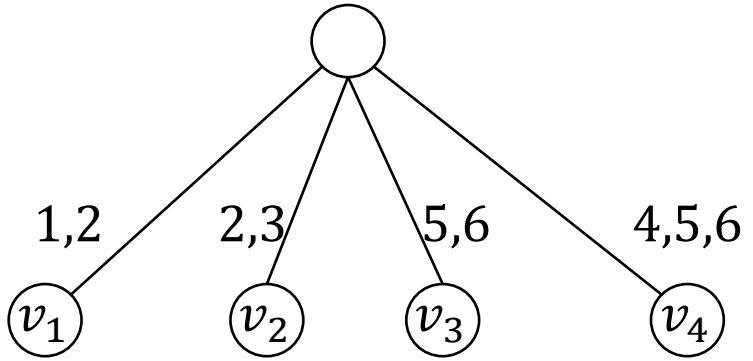

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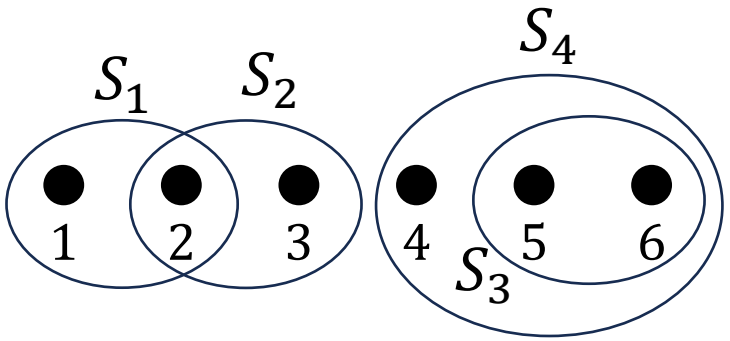

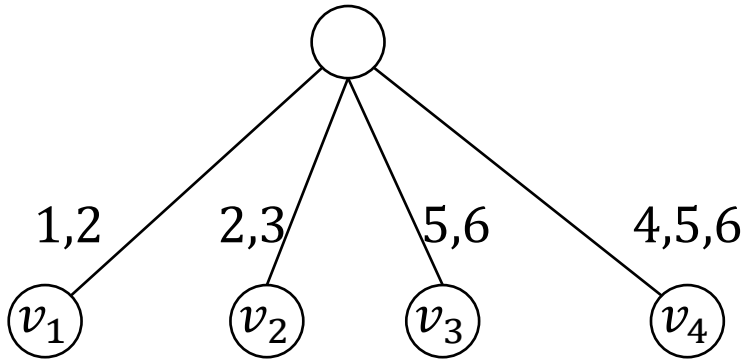

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$M = N$ $\Rightarrow$ Any oracle for STATIC temporal TREES supporting MD queries in worst-case time t requires $\widetilde{\Omega}(M^2/t^2)$ space…

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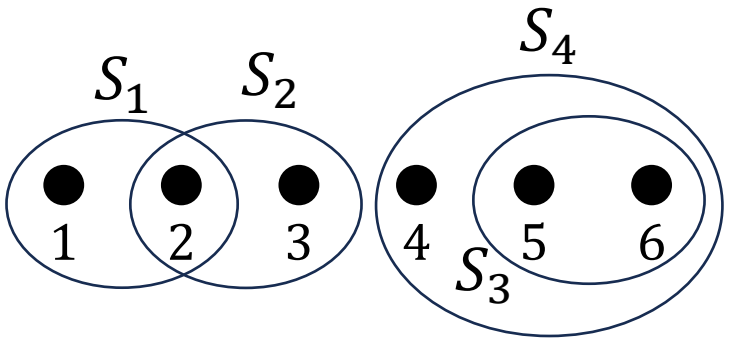

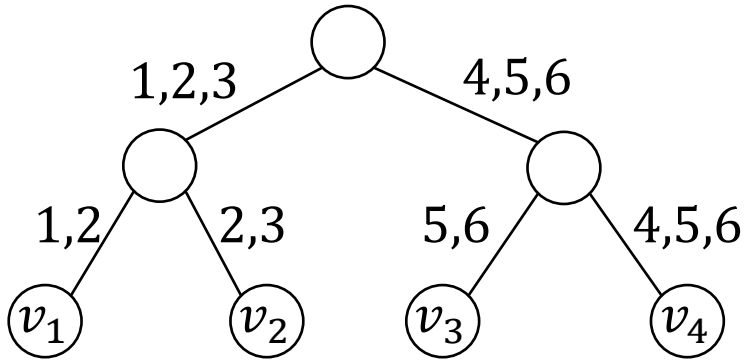

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$M = O(N \log N) \Rightarrow$ Any oracle for STATIC temporal TREES supporting MD queries in worst-case time t requires $\widetilde{\Omega}(M^2/t^2)$ space even if the tree has max degree 3.

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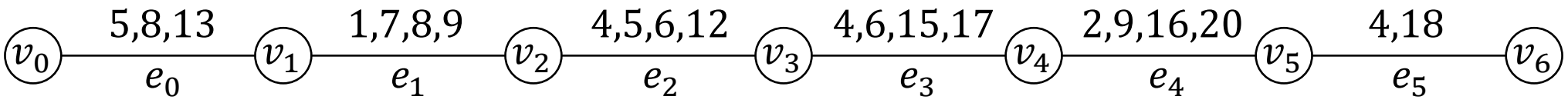
An almost successful attempt

IDEA 1: Model all useful *left-to-right* EA temporal paths.



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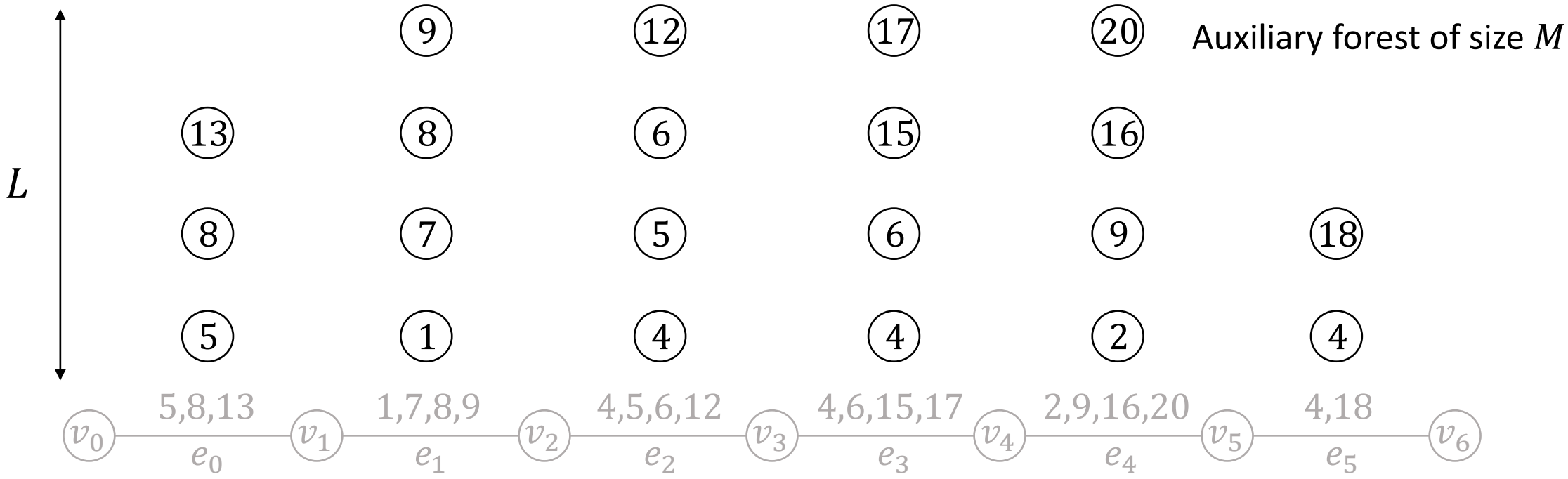

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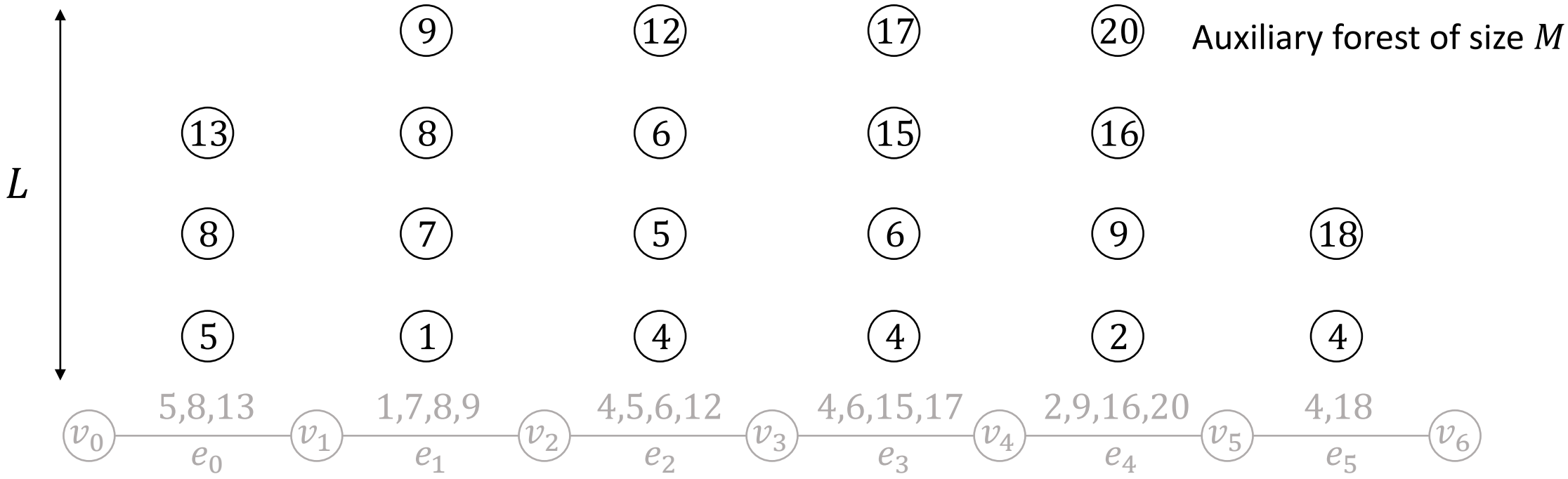

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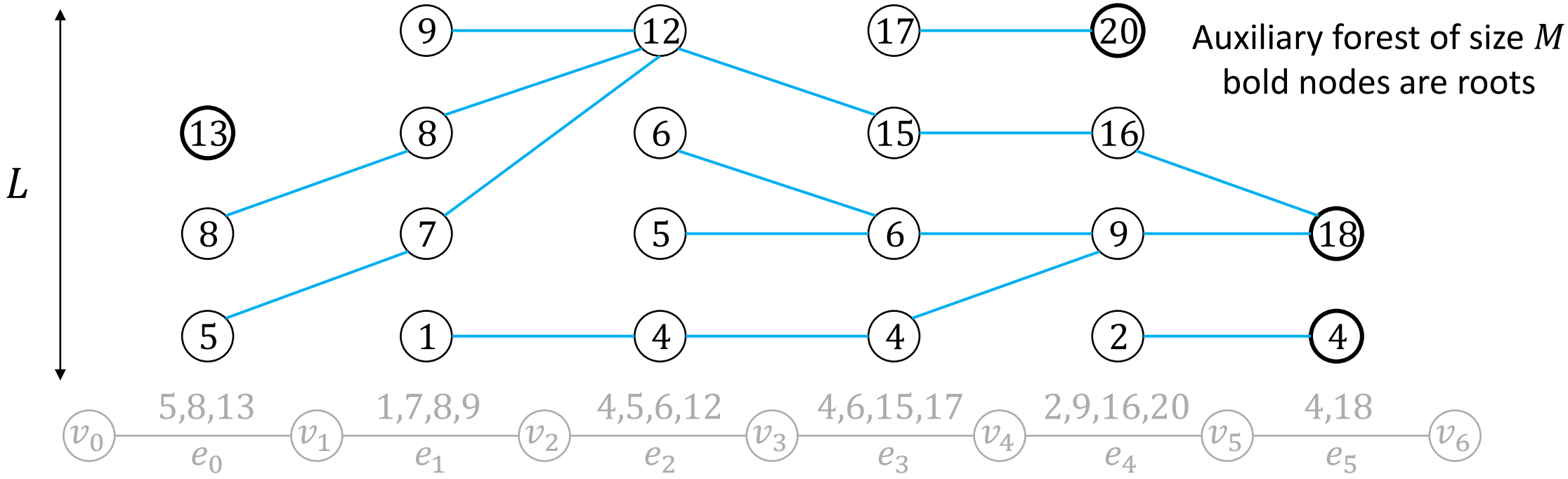
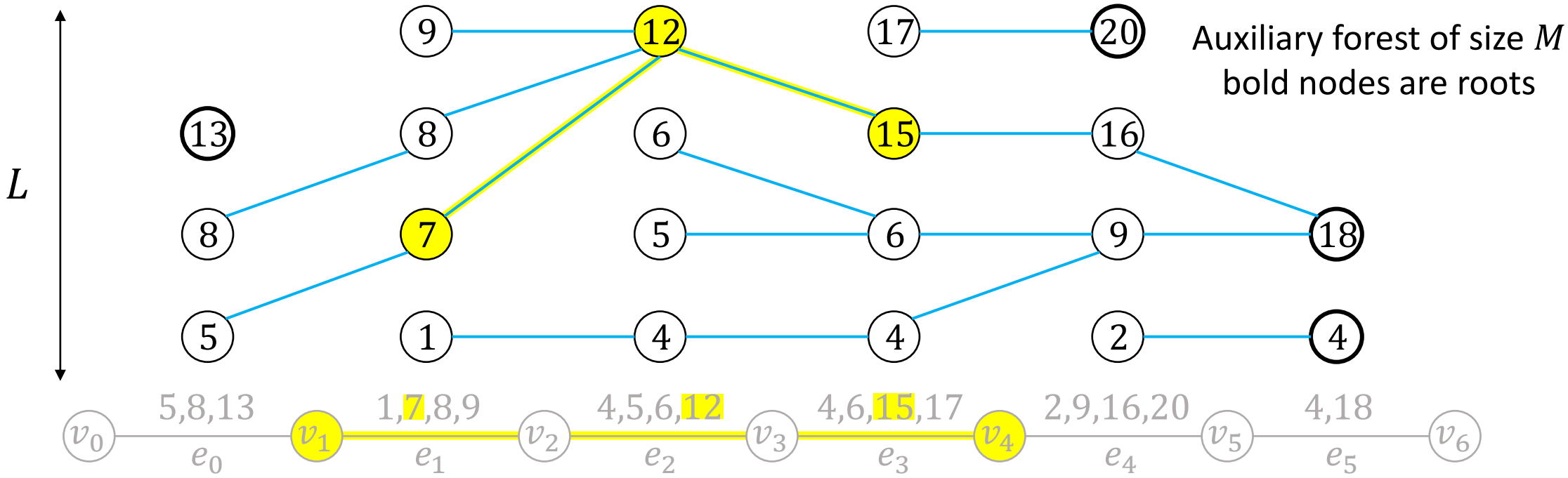


An almost successful attempt

$EA(v_i, v_j, d)$: Find non-strict successor of d in column i and return its $(j - i - 1)$th ancestor



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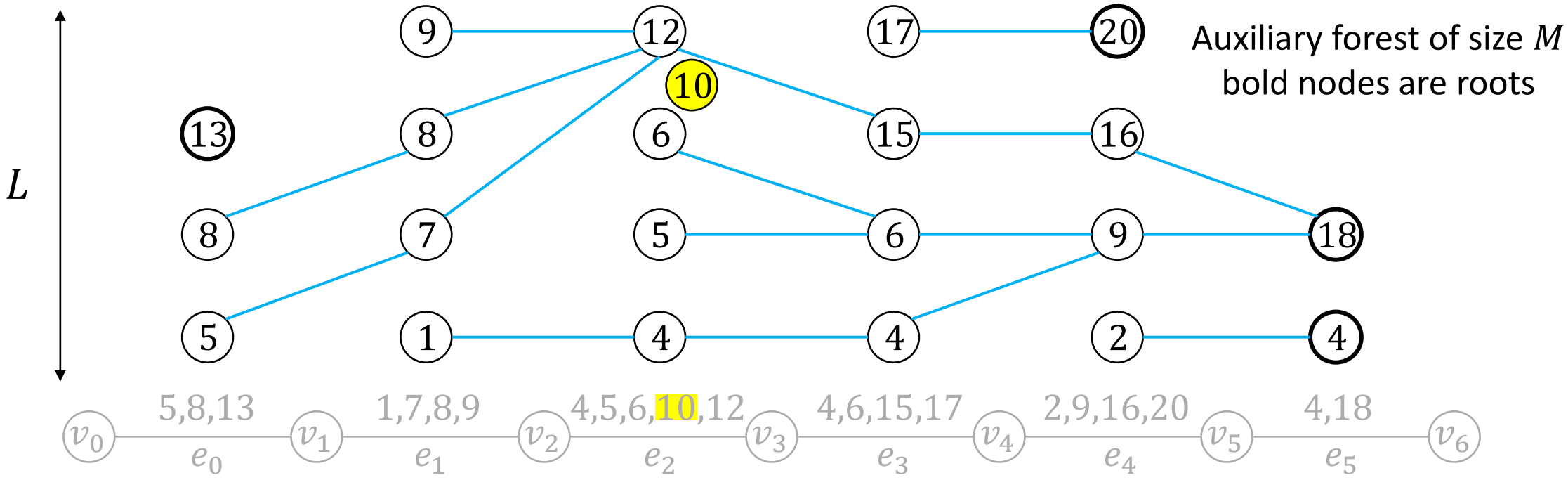

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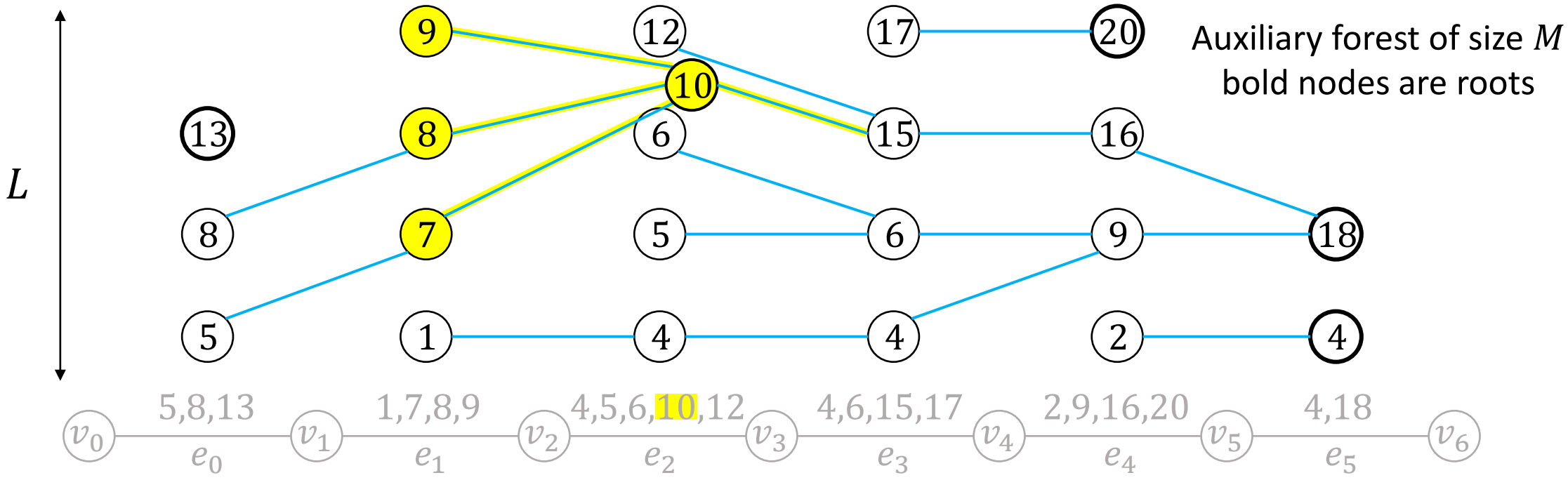
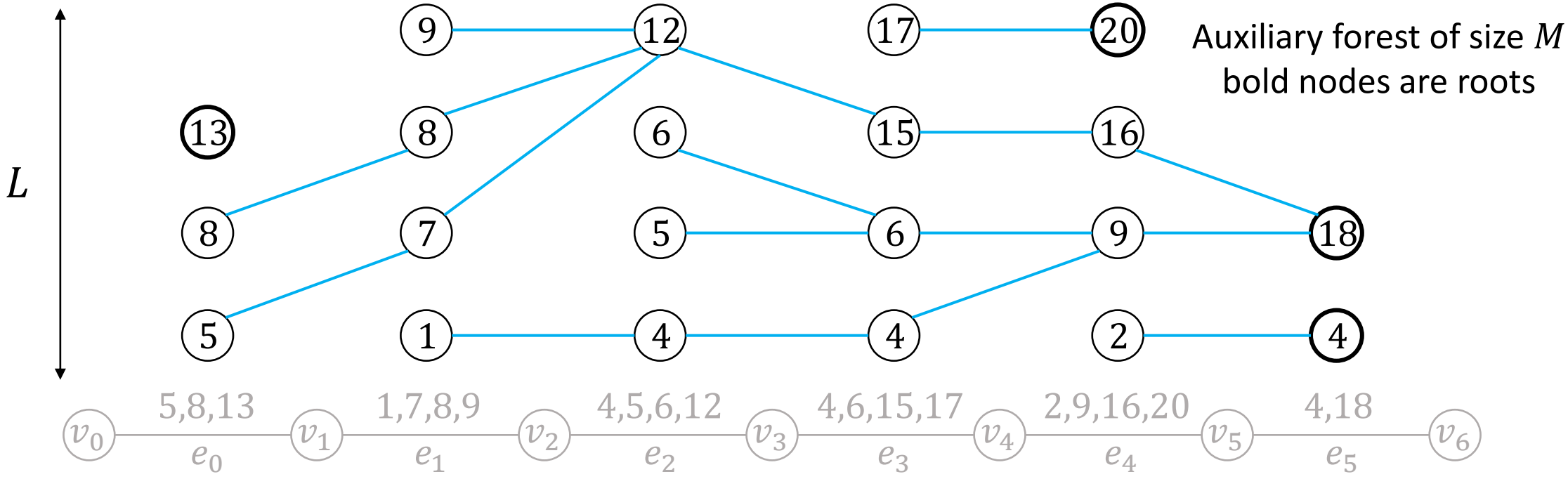


How to reduce node degrees

IDEA 2: Reduce the degree of nodes to a constant.



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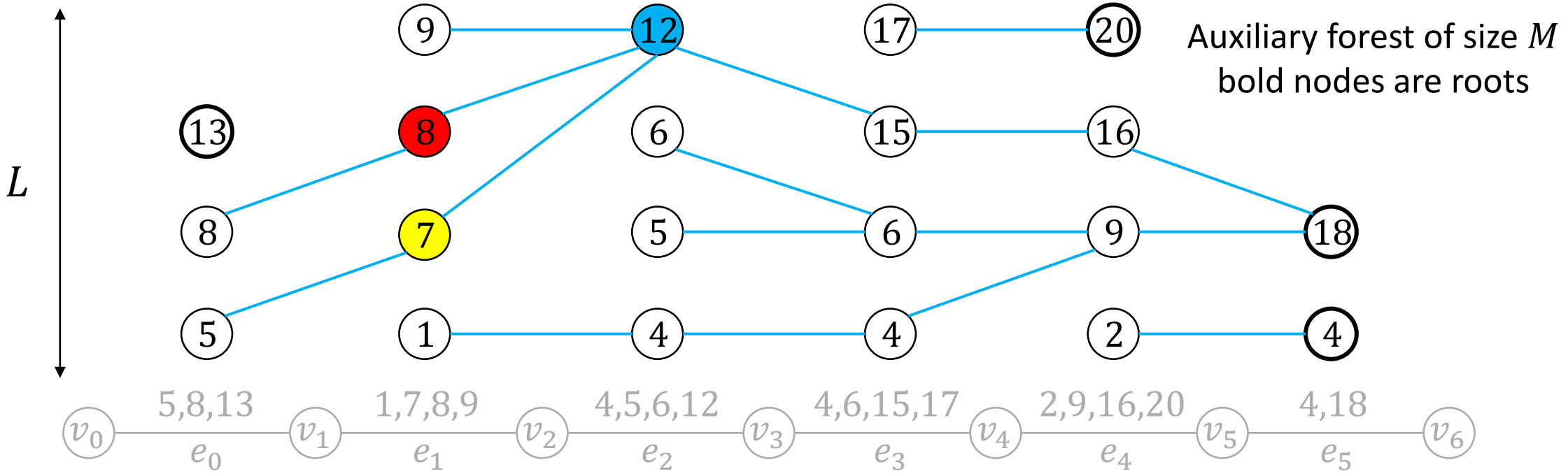

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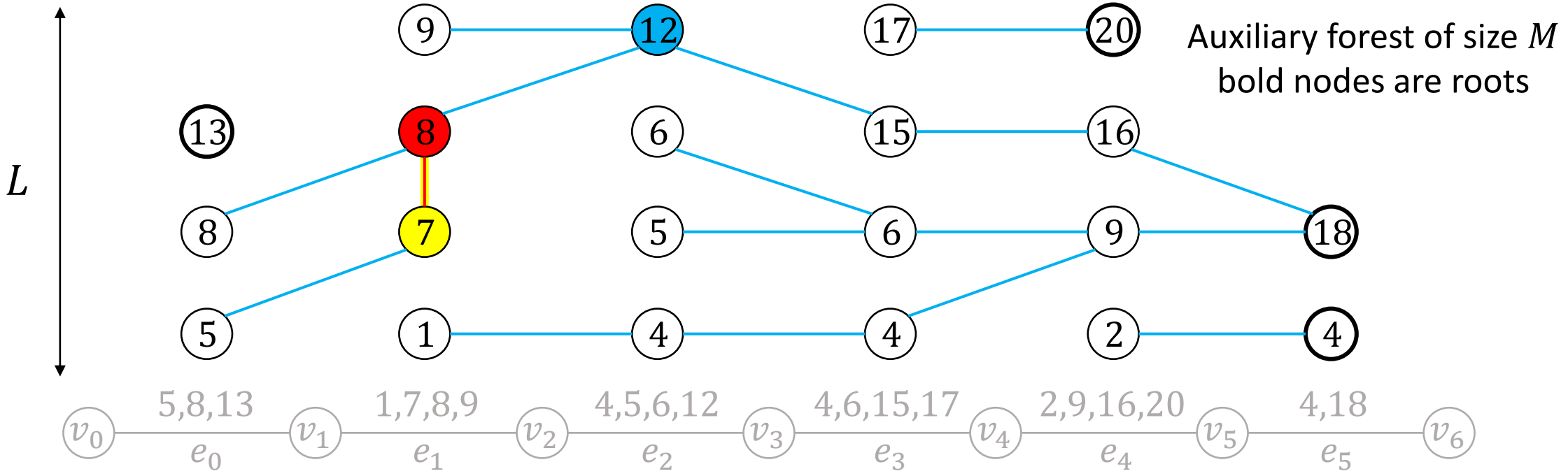

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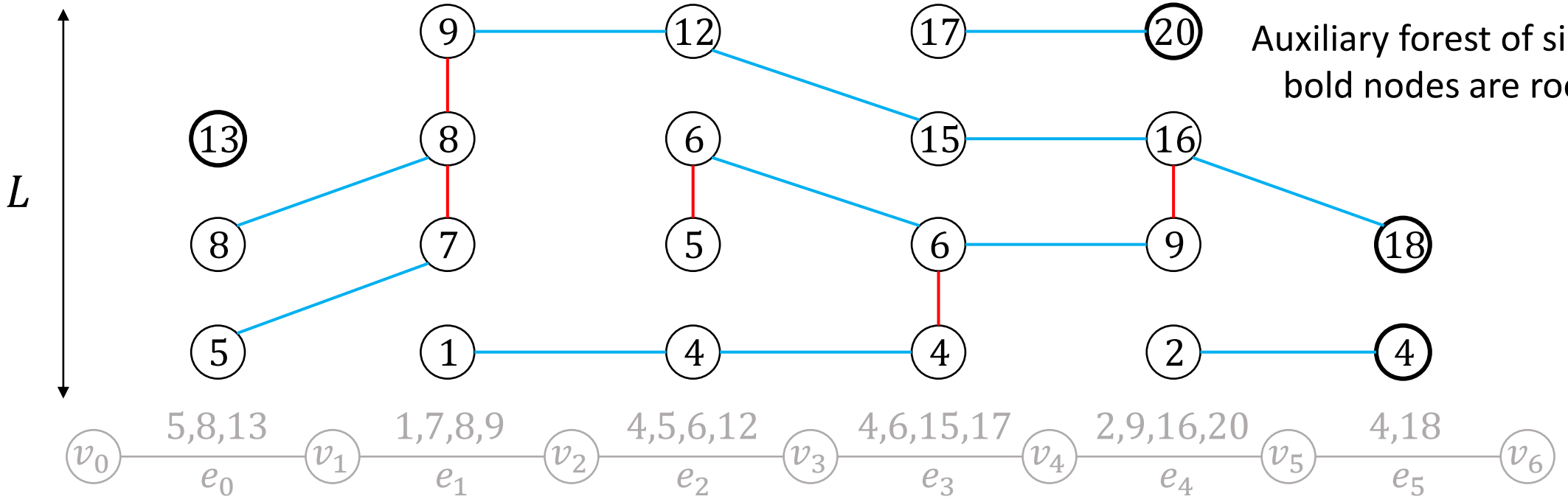
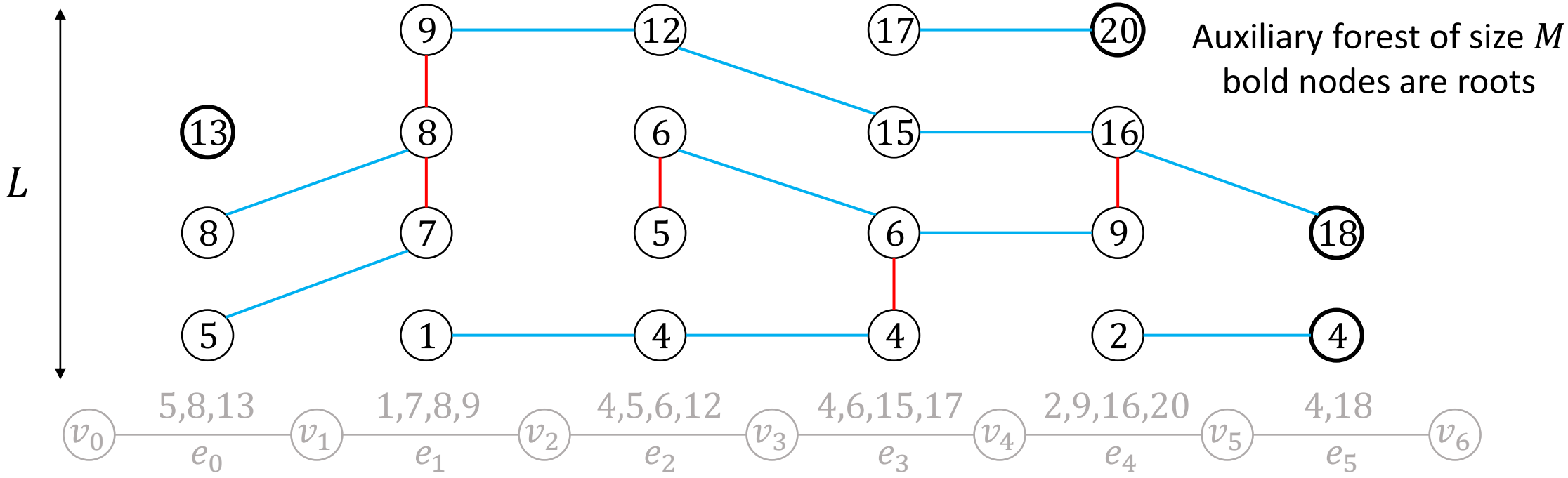


Fixing EA queries

IDEA 3: Add edge weights.

Blue edges have weight 1
Red edges have weight 0



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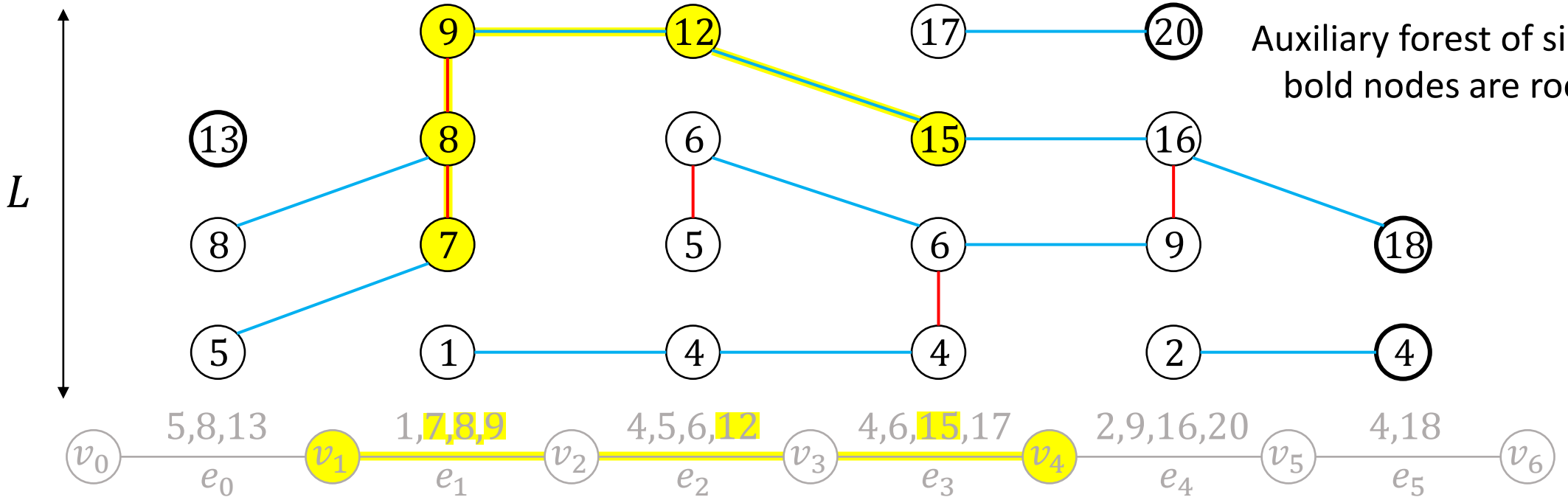


$EA(v_1, v_4, 5) = 15$

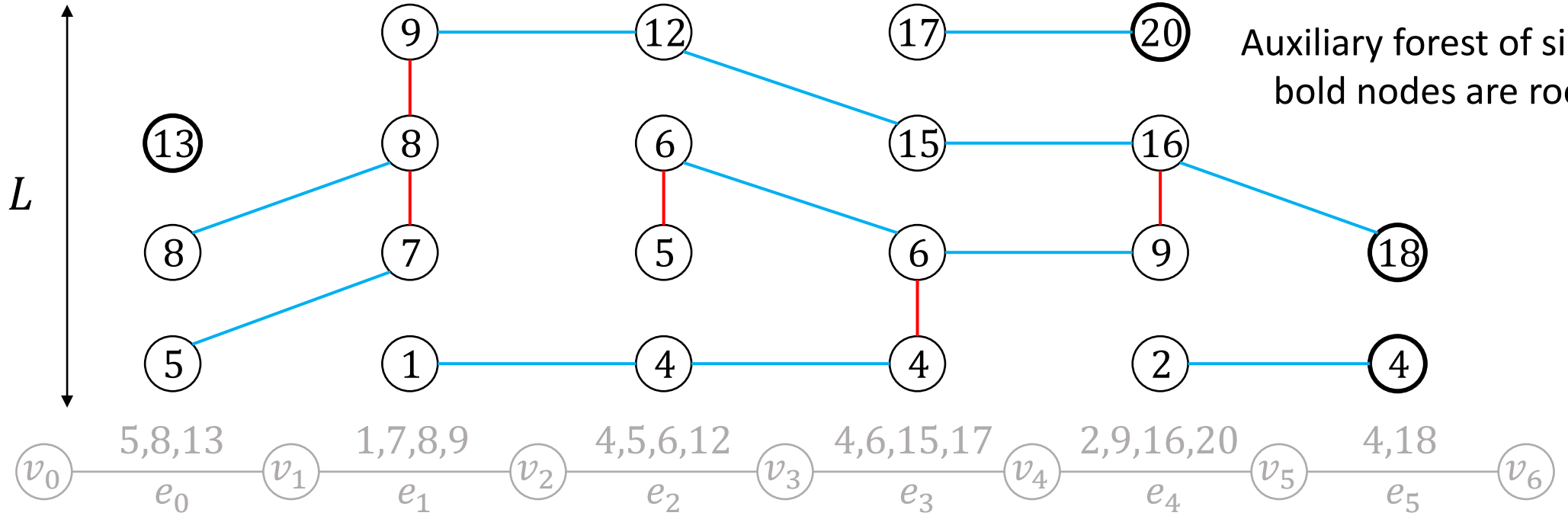
The oracle

A dictionary for each e_i to store its time labels

- total size is $O(M)$
- total building time is $O(M \log L)$
- insertions/deletions in $O(\log L)$ time
- predecessor/successor queries in $O(\log L)$ time

A top tree to store the auxiliary forest

- size is $O(M)$
- building time is $O(M)$
- node/edge insertions/deletions in $O(\log M)$ time
- weighted level-ancestor queries in $O(\log M)$ time



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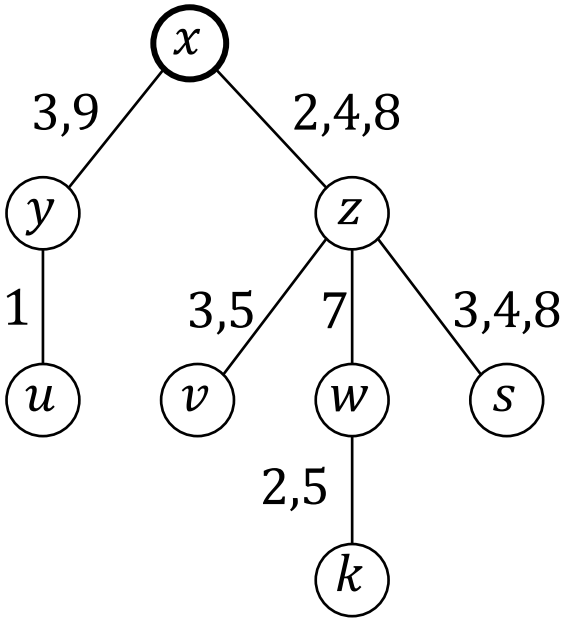
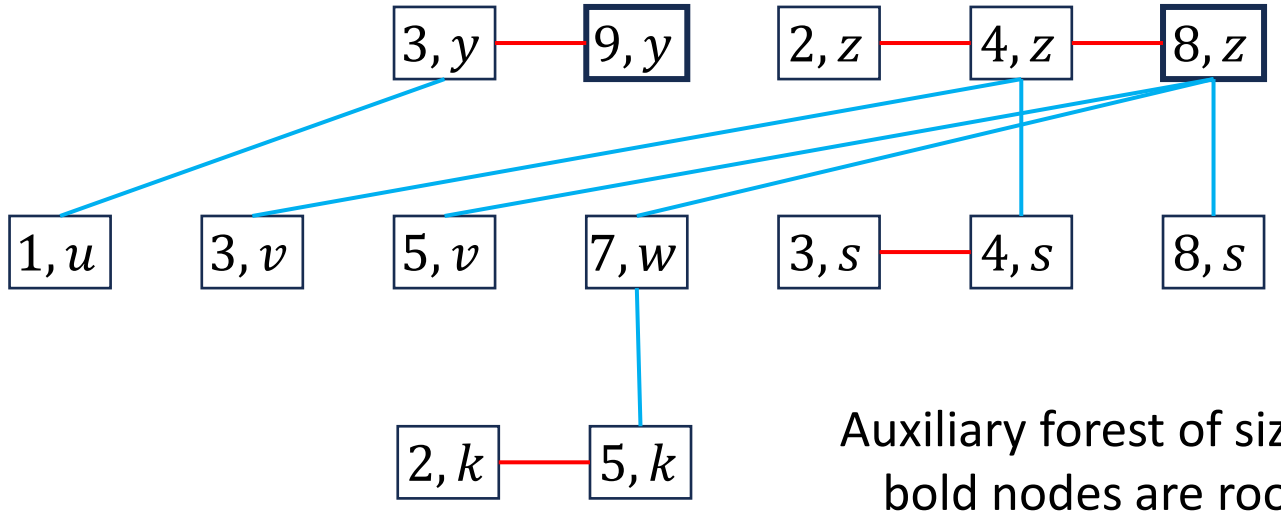
Oracle for dynamic temporal forests of rooted trees

RECYCLED IDEA: Model all useful *upwards* EA temporal paths.

It does not work as we create high-degree nodes.



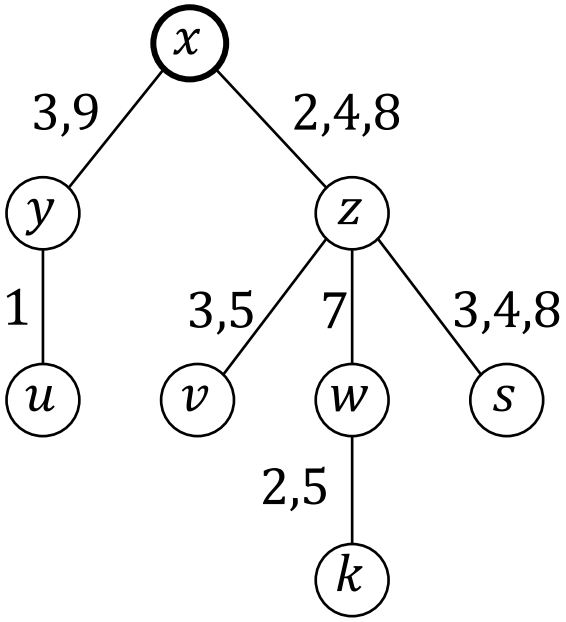


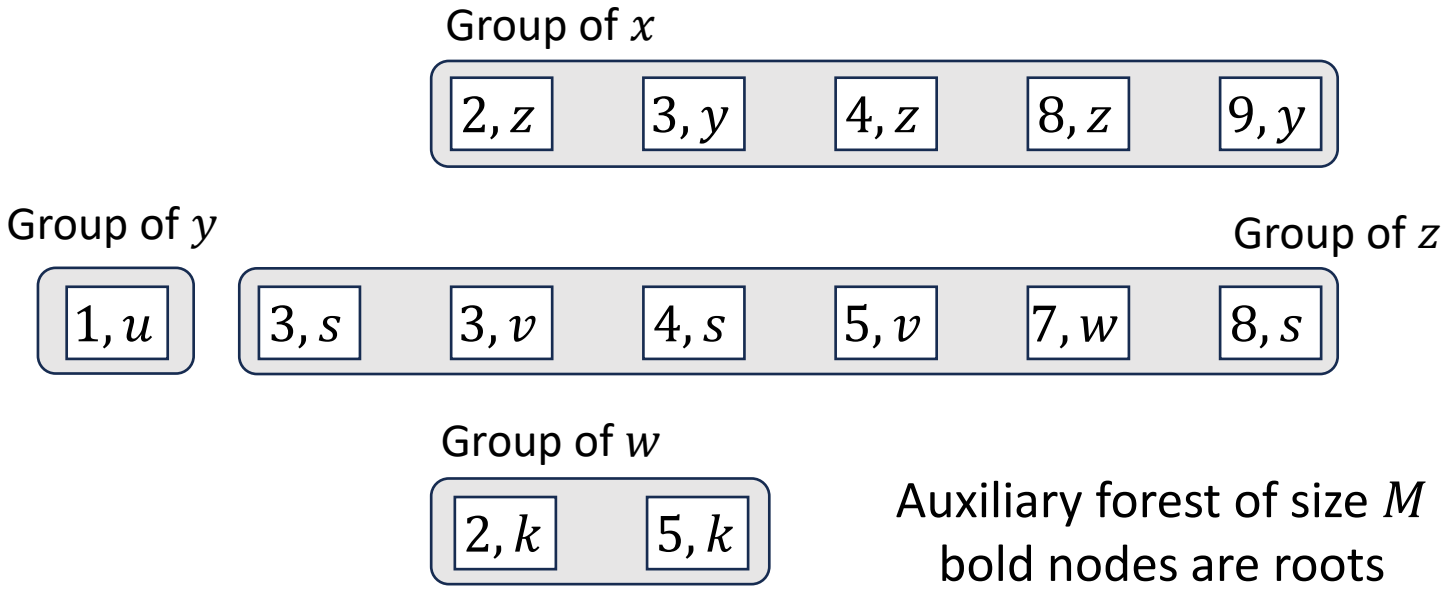
Auxiliary forest of size M
bold nodes are roots

Oracle for dynamic temporal forests of rooted trees

IDEA 2: Reduce the degree of nodes to a constant.

For each non-leaf v, group all nodes created by all edges branching from v





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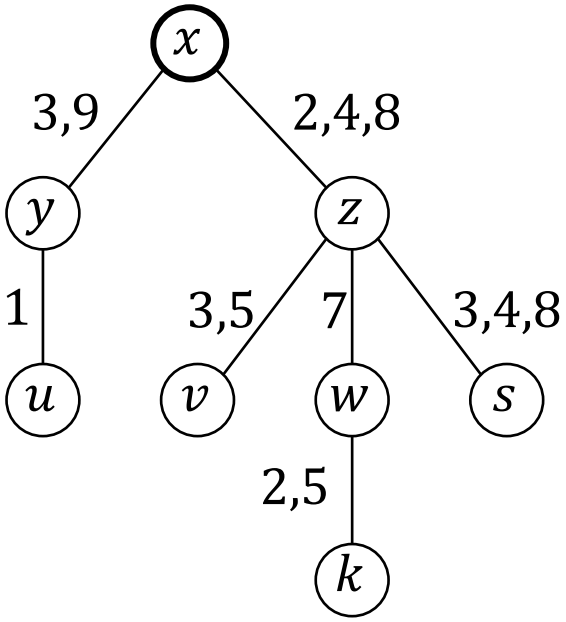
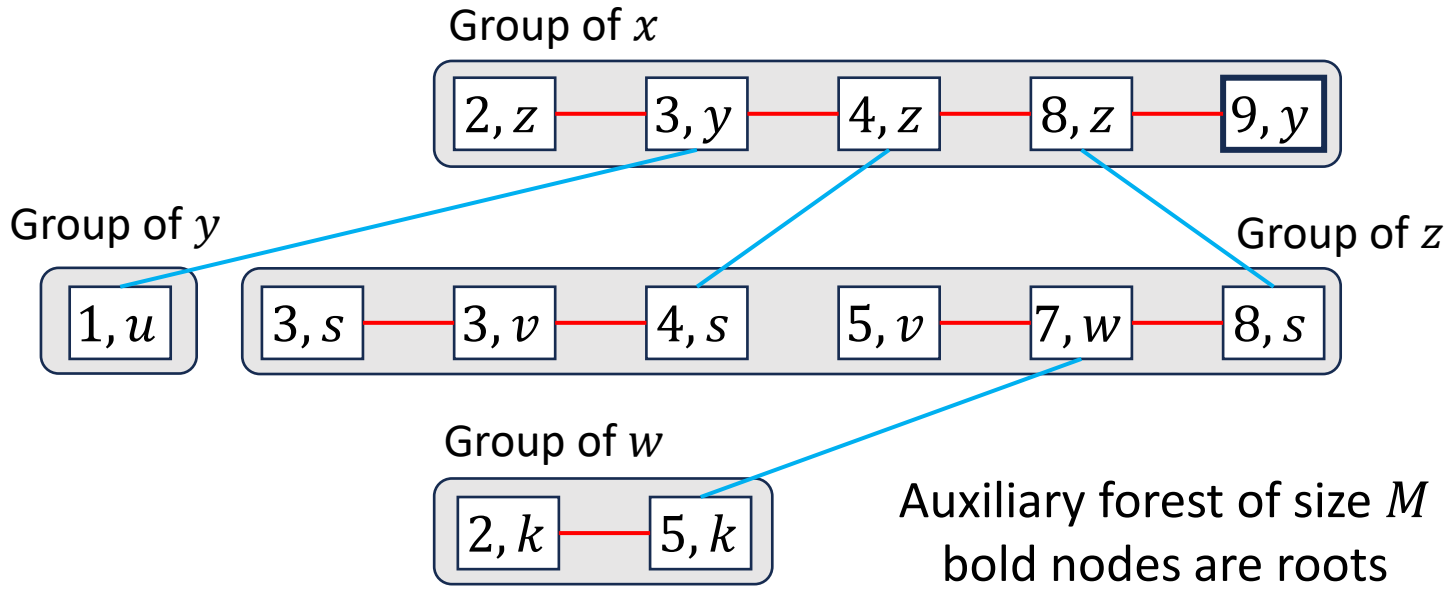
Candidate red parent of a node is computed in its group.

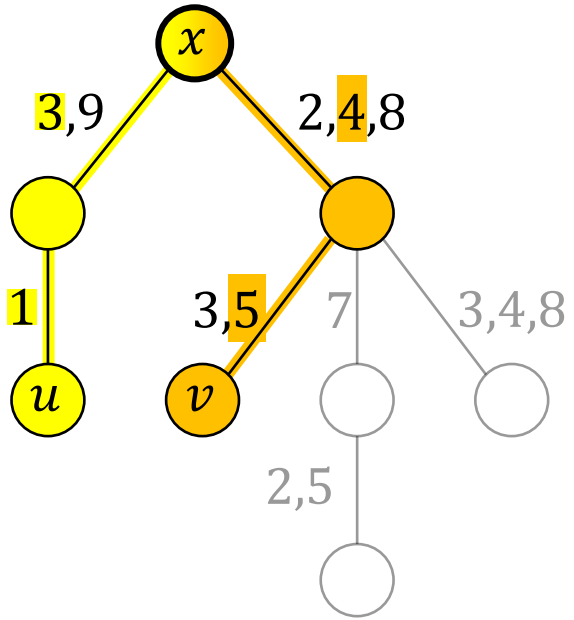




EA queries in $O(\log L \cdot \log M)$ time

$EA(u, v, d)$

1. $x = LCA(u, v)$
2. $d_x = EA(u, x, d)$ // upward query in $O(\log M)$ time
3. Return $EA(x, v, d_x)$ // downward query in $O(\log L \cdot \log M)$ time



$EA(u, v, 1) = 5$

- Upward query $EA(u, x, 1) = 3$
- Downward query $EA(x, v, 3) = 5$

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OPEN PROBLEM 1: Oracles for fully dynamic temporal forests with latencies supporting TR/EA/LD queries

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OPEN PROBLEM 1: Oracles for fully dynamic temporal forests with latencies supporting TR/EA/LD queries

OPEN PROBLEM 2: Better oracles for static paths supporting MD queries

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OPEN PROBLEM 5: Oracles supporting other types of queries (e.g., min travel time, min waiting time, etc.)

Thanks for your attention