Exploring temporal graphs

Joint work with:

Carla Groenland — *TU Delft*

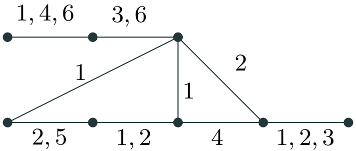
Lukas Michel — *University of Oxford*

Clément Rambaud — *Université Côte d'Azur*

Introduction

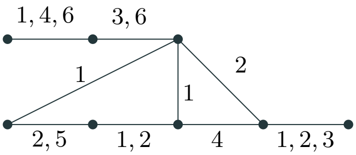
Introduction

A **temporal graph** G is a sequence of graphs $G = (G_t)_{t \in I}$ on the same vertex set indexed by an interval $I \subseteq \mathbb{N}$. The G_ts are called **snapshots** and $\cup_{t \in I} G_t$ is called the **underlying graph**.



Introduction

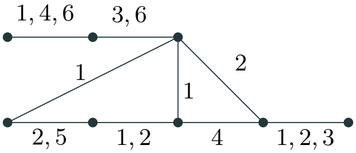
A **temporal graph** G is a sequence of graphs $G = (G_t)_{t \in I}$ on the same vertex set indexed by an interval $I \subseteq \mathbb{N}$. The G_ts are called **snapshots** and $\cup_{t \in I} G_t$ is called the **underlying graph**.



A **walk** in G is a sequence of vertices $v_1, v_2, \ldots, v_\ell$ such that either $v_i = v_{i+1}$ or $v_i v_{i+1} \in E(G_i)$.
An **exploration** of G is a walk that (starts on a prescribed vertex) and visits every vertex.

Introduction

A **temporal graph** G is a sequence of graphs $G = (G_t)_{t \in I}$ on the same vertex set indexed by an interval $I \subseteq \mathbb{N}$. The G_ts are called **snapshots** and $\cup_{t \in I} G_t$ is called the **underlying graph**.



A **walk** in G is a sequence of vertices $v_1, v_2, \ldots, v_\ell$ such that either $v_i = v_{i+1}$ or $v_i v_{i+1} \in E(G_i)$.
An **exploration** of G is a walk that (starts on a prescribed vertex) and visits every vertex.

What is the minimum length of an exploration?

Reachability

Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

Lemma

For any $u, v \in V(G)$, there is a walk of length $n - 1$ from u to v.

Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

$$F(u,t) = \{v \in V(G) \mid \text{there is a walk from } u \text{ to } v \text{ in } G_{[0,t]}\}$$

Lemma

For any $u, v \in V(G)$, there is a walk of length $n - 1$ from u to v.

Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

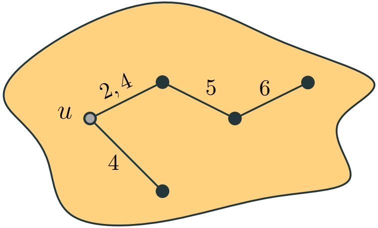
Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

$$F(u,t) = \{v \in V(G) \mid \text{there is a walk from } u \text{ to } v \text{ in } G_{[0,t]}\}$$

Lemma

For any $u, v \in V(G)$, there is a walk of length $n-1$ from u to v.



Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

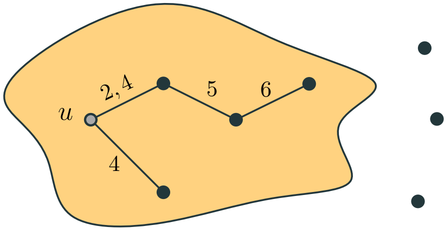
Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

$$F(u,t) = \{v \in V(G) \mid \text{there is a walk from } u \text{ to } v \text{ in } G_{[0,t]}\}$$

Lemma

For any $u, v \in V(G)$, there is a walk of length $n-1$ from u to v.



Reachability

A temporal graph $G = (G_t)_{t \in I}$ is **always-connected** if G_t is connected for all $t \in I$.

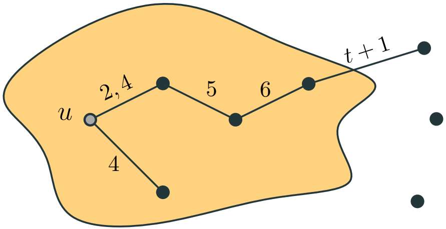
Lemma

Any always-connected temporal graph admits an exploration of length $O(n^2)$.

$$F(u,t) = \{v \in V(G) \mid \text{there is a walk from } u \text{ to } v \text{ in } G_{[0,t]}\}$$

Lemma

For any $u, v \in V(G)$, there is a walk of length $n-1$ from u to v.



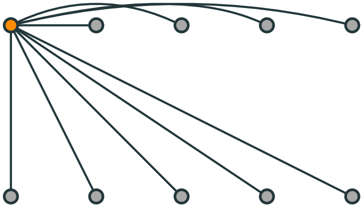
$$F(u,1) \subset F(u,2) \subset F(u,3) \subset \dots$$

The problem with high degree vertices

Is $O(n^2)$ the best we can do for always-connected temporal graphs?

The problem with high degree vertices

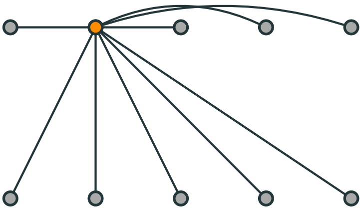
Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 1$

The problem with high degree vertices

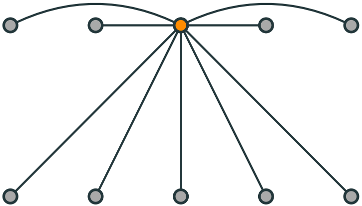
Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 2$

The problem with high degree vertices

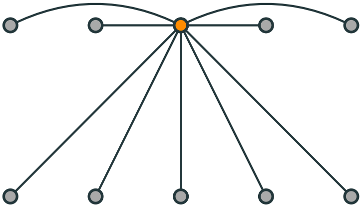
Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 3$

The problem with high degree vertices

Is $O(n^2)$ the best we can do for always-connected temporal graphs?



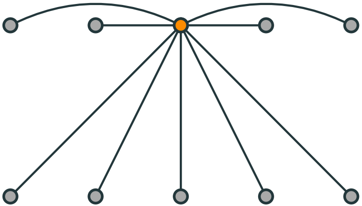
$t = 3$

Lemma

There exists G always-connected admitting no exploration in less than $n^2/4$ steps.

The problem with high degree vertices

Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 3$

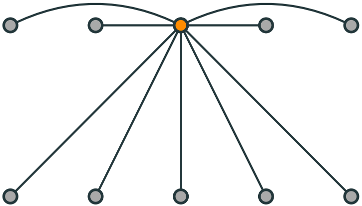
Lemma

There exists G always-connected admitting no exploration in less than $n^2/4$ steps.

Under what condition can we break the $\Theta(n^2)$ upper bound?

The problem with high degree vertices

Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 3$

Lemma

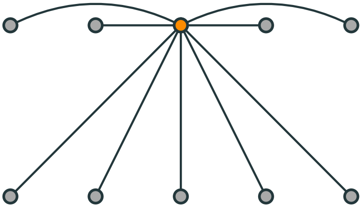
There exists G always-connected admitting no exploration in less than $n^2/4$ steps.

Under what condition can we break the $\Theta(n^2)$ upper bound?

High degree vertices .

The problem with high degree vertices

Is $O(n^2)$ the best we can do for always-connected temporal graphs?



$t = 3$

Lemma

There exists G always-connected admitting no exploration in less than $n^2/4$ steps.

Under what condition can we break the $\Theta(n^2)$ upper bound?

High degree vertices   Underlying graph is dense.

What if we restrict the underlying graph?

What if we restrict the underlying graph?

Theorem (Erlebach, Hoffman, Kammer 15)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{9/5})$ *steps.*

Theorem (Adamson, Gusev, Malyshev, Zamaraev 22)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{7/4} \log n)$ *steps.*

What if we restrict the underlying graph?

Theorem (Erlebach, Hoffman, Kammer 15)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{9/5})$ *steps.*

Theorem (Adamson, Gusev, Malyshev, Zamaraev 22)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{7/4} \log n)$ *steps.*

Theorem (Erlebach, Hoffman, Kammer 15)

If the underlying graph has **treewidth k** *then there exists an exploration in* $O(k^{3/2} n^{3/2} \log n)$ *steps.*

Theorem (Adamson, Gusev, Malyshev, Zamaraev 22)

If the underlying graph has **treewidth k** *then there exists an exploration in* $O(kn^{3/2} \log n)$ *steps.*

What if we restrict the underlying graph?

Theorem (Erlebach, Hoffman, Kammer 15)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{9/5})$ *steps.*

Theorem (Adamson, Gusev, Malyshev, Zamaraev 22)

If the underlying graph is **planar** *then there exists an exploration in* $O(n^{7/4}\log n)$ *steps.*

Theorem (Erlebach, Hoffman, Kammer 15)

If the underlying graph has **treewidth k** *then there exists an exploration in* $O(k^{3/2}n^{3/2}\log n)$ *steps.*

Theorem (Adamson, Gusev, Malyshev, Zamaraev 22)

If the underlying graph has **treewidth k** *then there exists an exploration in* $O(kn^{3/2}\log n)$ *steps.*

And more results: cactus, a $2 \times n$ grid, path, etc.

What if we restrict the snapshots?

What if every snapshot has bounded degree?

What if we restrict the snapshots?

What if every snapshot has bounded degree?

Theorem (Erlebach, Spooner 18)

If all snapshots have bounded degree to all then there is an exploration of length $O(n^2 \log d / \log n)$.

What if we restrict the snapshots?

What if every snapshot has bounded degree?

Theorem (Erlebach, Spooner 18)

If all snapshots have bounded degree to all then there is an exploration of length $O(n^2 \log d / \log n)$.

Theorem (Erleback, Kammer, Luo, Sajenko, Spooner 19)

If all snapshots have bounded degree, then there is an exploration of length $O(n^{7/4})$.

What if we restrict the snapshots?

What if every snapshot has bounded degree?

Theorem (Erlebach, Spooner 18)

If all snapshots have bounded degree to all then there is an exploration of length $O(n^2 \log d / \log n)$.

Theorem (Erleback, Kammer, Luo, Sajenko, Spooner 19)

If all snapshots have bounded degree, then there is an exploration of length $O(n^{7/4})$.

Theorem (AGMZ 22)

There exists G where every snapshot is a path, and the shortest exploration has length $\Theta(n \log n)$.

Theorem (EHK15)

For any $d \leq n$, there exists G where every snapshot has maximum degree d, and the shortest exploration has length $\Theta(dn)$.

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.*

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length* $O(n^{3/2}\sqrt{d \log n})$.

The average temporal maximum degree of a temporal graph G is:

$$\frac{1}{n} \sum_{u \in V(G)} \max_{t \in I} d_{G_t}(u).$$

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length* $O(n^{3/2}\sqrt{d \log n})$.

The average temporal maximum degree of a temporal graph G is:

$$\frac{1}{n} \sum_{u \in V(G)} \max_{t \in I} d_{G_t}(u).$$

G_t max degree d		
$O_d(n^{7/4})$ $\downarrow$ $O(n^{3/2}\sqrt{d \log n})$		

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.*

The average temporal maximum degree of a temporal graph G is:

$$\frac{1}{n} \sum_{u \in V(G)} \max_{t \in I} d_{G_t}(u).$$

G_t max degree d	planar	treewidth k	
$O_d(n^{7/4})$	$O(n^{7/4} \log n)$	$O(kn^{3/2} \log n)$	
$\downarrow$	$\downarrow$	$\downarrow$	
$O(n^{3/2}\sqrt{d \log n})$	$O(n^{3/2}\sqrt{\log n})$	$O(n^{3/2}\sqrt{k \log n})$	

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.*

The average temporal maximum degree of a temporal graph G is:

$$\frac{1}{n} \sum_{u \in V(G)} \max_{t \in I} d_{G_t}(u).$$

G_t max degree d	planar	treewidth k	K_t-minor free
$O_d(n^{7/4})$	$O(n^{7/4} \log n)$	$O(kn^{3/2} \log n)$	$O(n^2)$
$\downarrow$	$\downarrow$	$\downarrow$	
$O(n^{3/2}\sqrt{d \log n})$	$O(n^{3/2}\sqrt{\log n})$	$O(n^{3/2}\sqrt{k \log n})$	

One parameter to rule them all

Theorem (B., Groenland, Michel, Rambaud 25+)

If G has **average temporal maximum degree** *at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.*

The average temporal maximum degree of a temporal graph G is:

$$\frac{1}{n} \sum_{u \in V(G)} \max_{t \in I} d_{G_t}(u).$$

G_t max degree d	planar	treewidth k	K_t-minor free
$O_d(n^{7/4})$	$O(n^{7/4} \log n)$	$O(kn^{3/2} \log n)$	$O(n^2)$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$O(n^{3/2}\sqrt{d \log n})$	$O(n^{3/2}\sqrt{\log n})$	$O(n^{3/2}\sqrt{k \log n})$	$t^{1/2+o(1)} n^{3/2} \sqrt{\log n}$

Overall approach

Theorem (B., Groenland, Michel, Rambaud 25+)

For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.

Overall approach

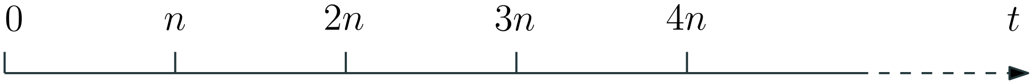
Theorem (B., Groenland, Michel, Rambaud 25+)

For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.

Overall approach

Theorem (B., Groenland, Michel, Rambaud 25+)

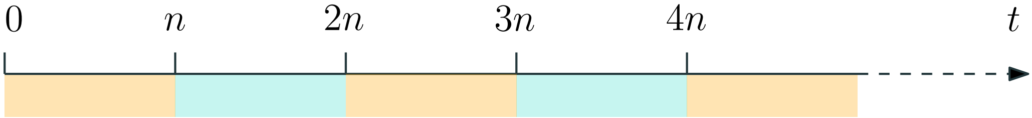
For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.



Overall approach

Theorem (B., Groenland, Michel, Rambaud 25+)

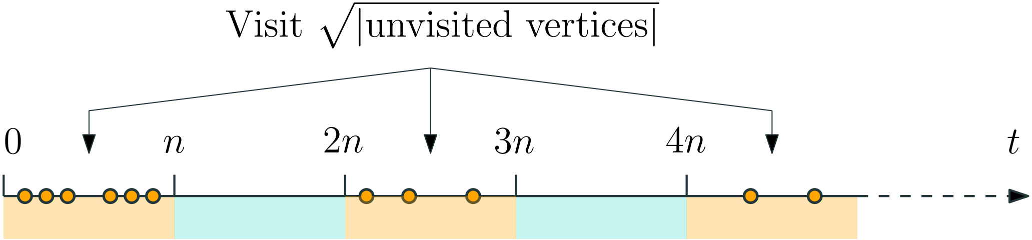
For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.



Overall approach

Theorem (B., Groenland, Michel, Rambaud 25+)

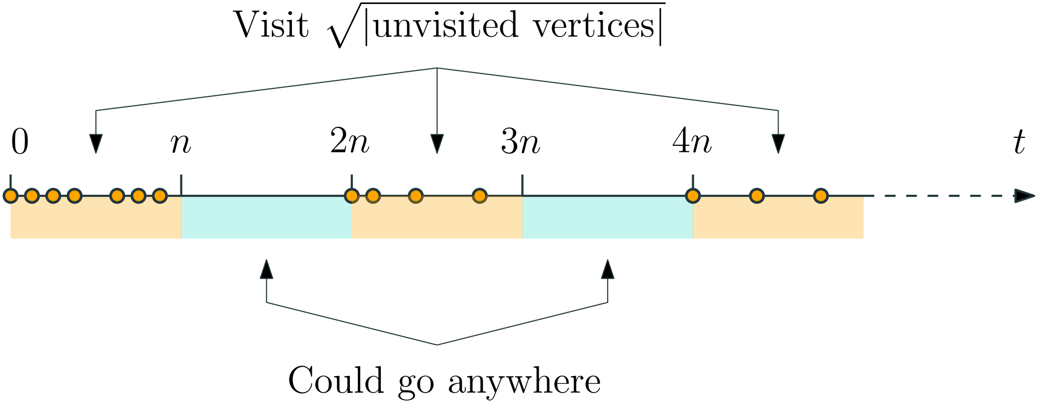
For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.



Overall approach

Theorem (B., Groenland, Michel, Rambaud 25+)

For any G such that all snapshots have degree at most d, then there is an exploration of length $O(n^{3/2}\sqrt{d \log n})$.



The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

The main Lemma

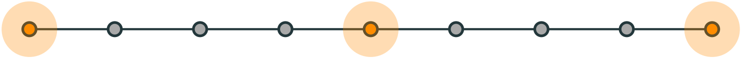
Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

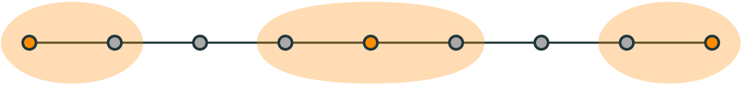
For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

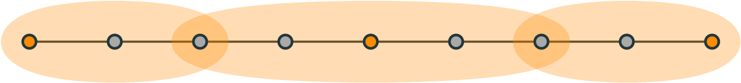
For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



The main Lemma

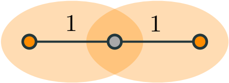
Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

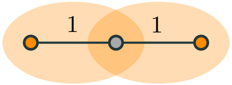
For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

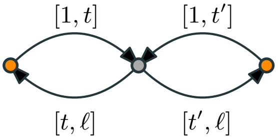


The main Lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.





Let ℓ be the min length of a walk between a pair in S

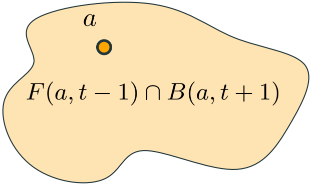
$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V :$ there is a temporal walk from a to w in $G_{[1,t]}\}$

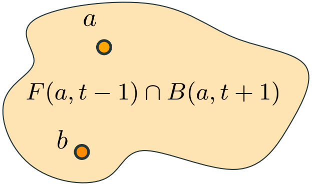
$B(a,t) = \{w \in V :$ there is a temporal walk from w to a in $G_{[t,\ell]}\}$



Let ℓ be the min length of a walk between a pair in S

$$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$$

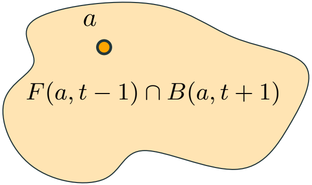
$$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$$



Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V :$ there is a temporal walk from a to w in $G_{[1,t]}\}$

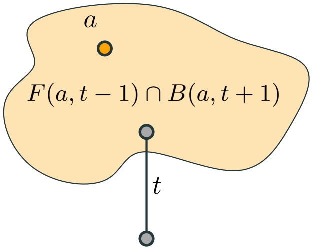
$B(a,t) = \{w \in V :$ there is a temporal walk from w to a in $G_{[t,\ell]}\}$



Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

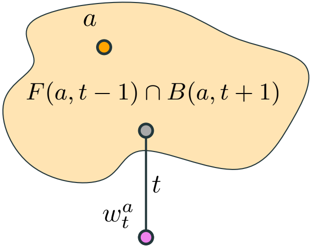
$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

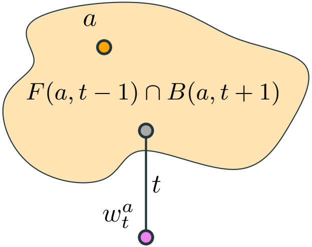
$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$

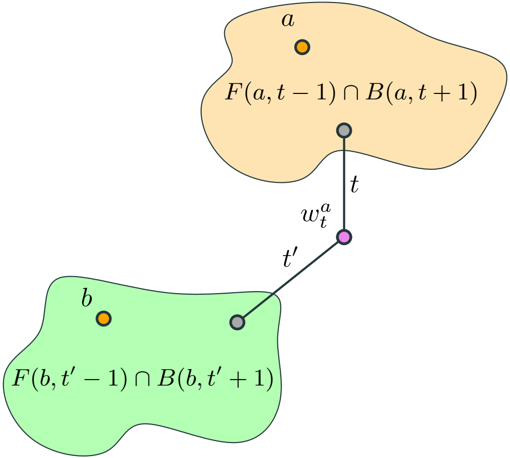


(1) If $w_t^a = w_{t'}^b$

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$

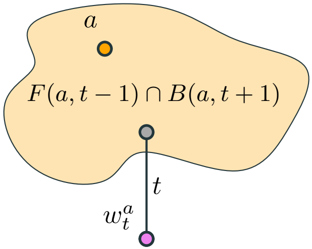


(1) If $w_t^a = w_{t'}^b$

Let ℓ be the min length of a walk between a pair in S

$$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$$

$$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$$

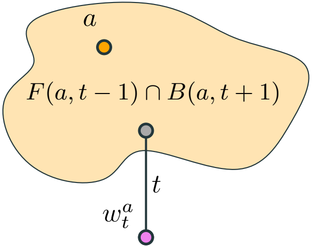


(1) If $w_t^a = w_{t'}^b$ $\to$ done!

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



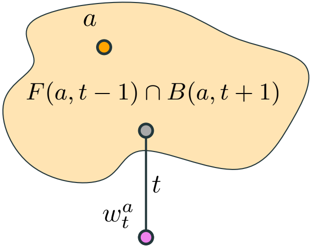
(1) If $w_t^a = w_{t'}^b \;\rightarrow$ done!

(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a$

Let ℓ be the min length of a walk between a pair in S

$$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$$

$$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$$



(1) If $w_t^a = w_{t'}^b \;\rightarrow$ done!

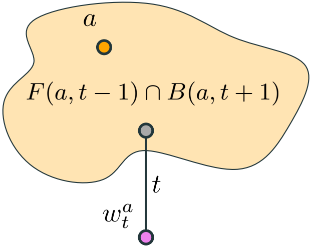
(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a$

$\in F(a,t_1)$ $\qquad\qquad$ $\in B(a,t_2)$

Let ℓ be the min length of a walk between a pair in S

$F(a, t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a, t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



(1) If $w_t^a = w_{t'}^b \rightarrow$ done!

(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a$

$\swarrow \qquad \searrow$

$\in F(a, t_1) \qquad \in B(a, t_2)$

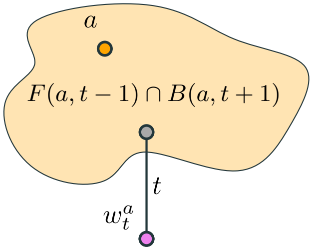
$\searrow \qquad \swarrow$

$\in F(u, t-1) \cap B(u, t+1)$

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



(1) If $w_t^a = w_{t'}^b \rightarrow$ done!

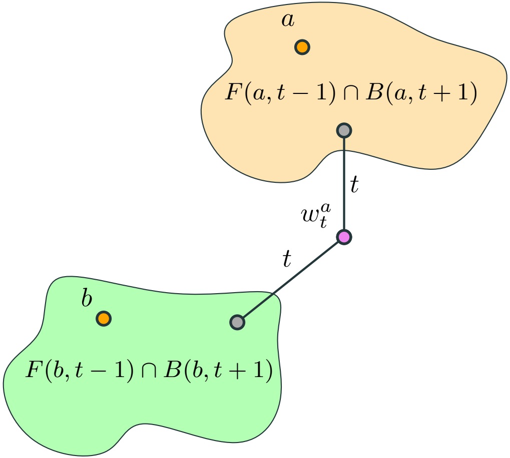
(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a \quad \rightarrow$ impossible

(3) If $w_t^a = w_t^b = w_t^c = \dots$

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



(1) If $w_t^a = w_{t'}^b$ $\rightarrow$ done!

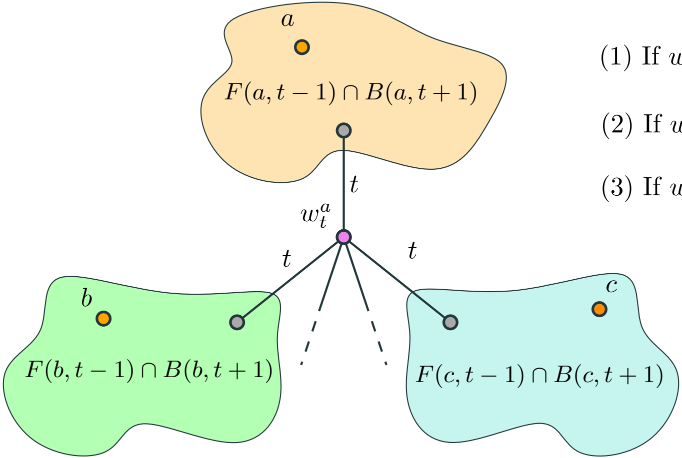
(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a$ $\rightarrow$ impossible

(3) If $w_t^a = w_t^b = w_t^c = \ldots$

Let ℓ be the min length of a walk between a pair in S

$$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$$

$$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$$



(1) If $w_t^a = w_{t'}^b \rightarrow$ done!

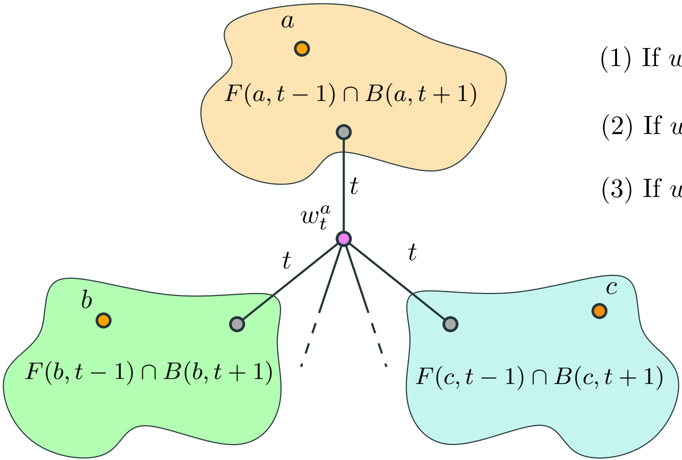
(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a \rightarrow$ impossible

(3) If $w_t^a = w_t^b = w_t^c = \dots$

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$



(1) If $w_t^a = w_{t'}^b \rightarrow$ done!

(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a \rightarrow$ impossible

(3) If $w_t^a = w_t^b = w_t^c = \ldots \rightarrow$ at most d

Let ℓ be the min length of a walk between a pair in S

$F(a,t) = \{w \in V : \text{there is a temporal walk from } a \text{ to } w \text{ in } G_{[1,t]}\}$

$B(a,t) = \{w \in V : \text{there is a temporal walk from } w \text{ to } a \text{ in } G_{[t,\ell]}\}$

(1) If $w_t^a = w_{t'}^b$ $\rightarrow$ done!

(2) If $w_{t_1}^a = w_t^a = w_{t_2}^a$ $\rightarrow$ impossible

(3) If $w_t^a = w_t^b = w_t^c = \dots$ $\rightarrow$ at most d

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

Theorem (B., Groenland, Michel, Rambaud 25+)

If all snapshots have degree at most Δ, then there is an exploration of length $O(n^{3/2}\sqrt{\Delta \log n})$.

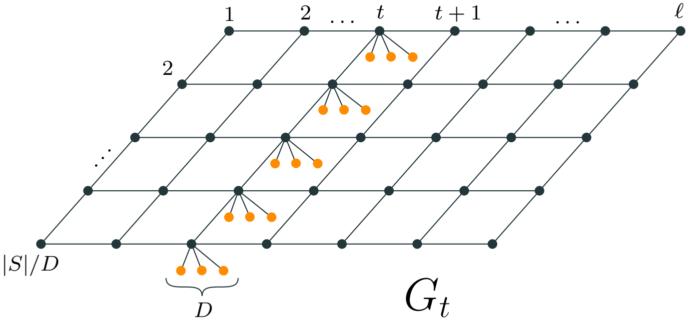
Theorem (B., Groenland, Michel, Rambaud 25+)

If the average degree of the underlying temporal graph is at most Δ, then there is an exploration of length $O(n^{3/2}\sqrt{\Delta \log n})$.

Tightness of the lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

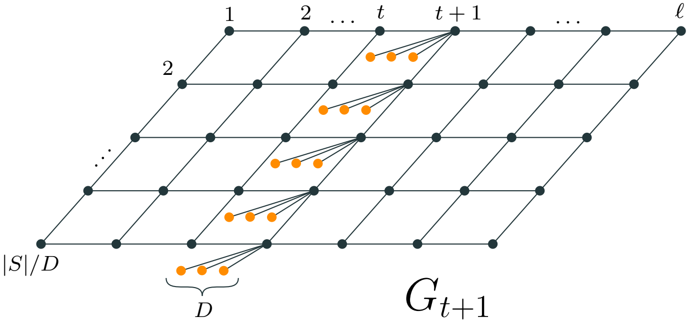
For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



Tightness of the lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

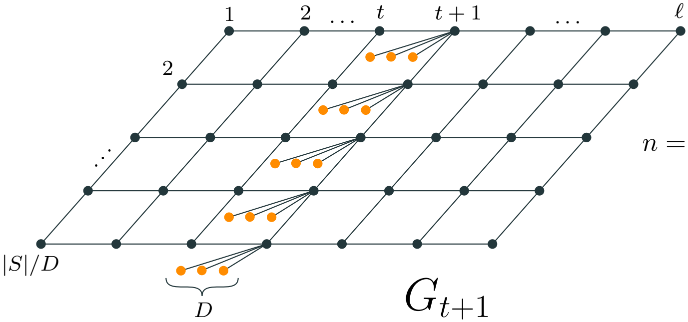
For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



Tightness of the lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.

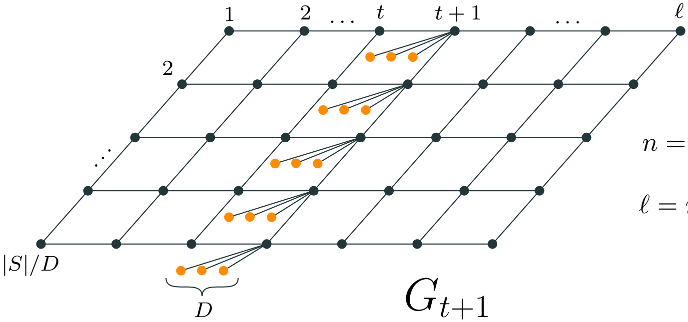


$n = |S| + \ell \cdot |S|/D$

Tightness of the lemma

Lemma [B., Groenland,Michel,Rambaud 25+]

For any set of vertices $S \subseteq V(G)$, there exists a walk between two distinct vertices $u, v \in S$ of length at most $2\Delta n/|S|$.



$$n = |S| + \ell \cdot |S|/D$$

$$\ell = nD/|S| - D$$

Open questions

Question

Is the *average temporal maximum degree* the right parameter to consider?

Open questions

Question

Is the *average temporal maximum degree* the right parameter to consider?

Question

Is there a function $f = \omega(1)$ such that for all integers $n, \Delta \geq 1$, there is a temporal graph $G = (G_t)_{t \in \mathbb{N}}$ on n vertices with average temporal maximum degree at most Δ, yet the length of a shortest temporal exploration of G is at least $f(\Delta) \cdot n \log n$?

Open questions

Question

Is the *average temporal maximum degree* the right parameter to consider?

Question

Is there a function $f = \omega(1)$ such that for all integers $n, \Delta \geq 1$, there is a temporal graph $G = (G_t)_{t \in \mathbb{N}}$ on n vertices with average temporal maximum degree at most Δ, yet the length of a shortest temporal exploration of G is at least $f(\Delta) \cdot n \log n$?

Question

What is the length of an exploration if all snapshots are expanders?

Open questions

Question

Is the *average temporal maximum degree* the right parameter to consider?

Question

Is there a function $f = \omega(1)$ such that for all integers $n, \Delta \geq 1$, there is a temporal graph $G = (G_t)_{t\in\mathbb{N}}$ on n vertices with average temporal maximum degree at most Δ, yet the length of a shortest temporal exploration of G is at least $f(\Delta) \cdot n \log n$?

Question

What is the length of an exploration if all snapshots are expanders?

Thank you!

2 moves per time step

Question

Is the longuest exploration still $\Theta(n^2)$ we allow 2 moves per time step?

2 moves per time step

Question

Is the longest exploration still $\Theta(n^2)$ we allow 2 moves per time step?

Theorem [B., Groenland,Michel,Rambaud 25+][Erleback, Kammer, Luo, Sajenko, Spooner 19]

Our result applies and there exists an exploration of length $O(n^{3/2}\sqrt{\log n})$.