Temporalizing Multi-Digraphs via Linear-Size Balanced Bi-Trees

Laurent Viennot

joint work with :
Alkida Balliu, Filippo Brunelli, Pierluigi Crescenzi, Dennis Olivetti
and :
Stéphane Bessy, Stéphan Thomassé

Algorithmic Aspects of Temporal Graphs 2024

Outline

Introduction : temporal graphs

Problem : temporalization

An approximate solution

Main tool : a balanced-separator Lemma for digraphs

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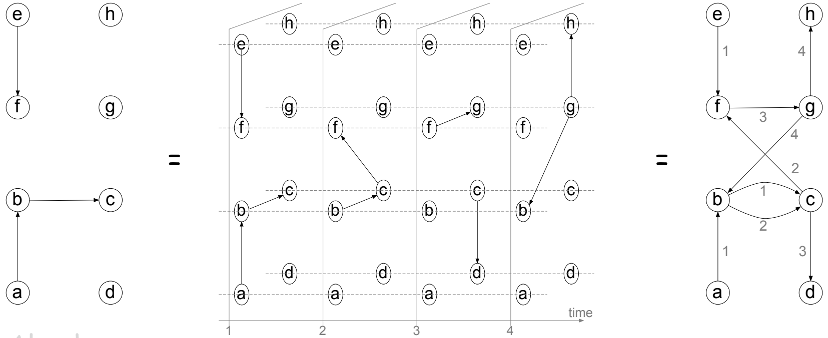
Temporal graphs (basic example)



Also known as :

- **time-dependent networks** [Cooke, Halsey 1966],
- **edge scheduled networks** [Berman 1996], **dynamic graphs** [Harary, Gupta 1997], **temporal networks** [Kempe, Kleinberg, Kumar 2002; Holme '15], **evolving graphs** [Bhadra, Ferreira '03],
- **time-varying graphs** [Casteigts, Flocchini, Quattrociocchi, Santoro 2012],
- **link streams** [Latapy, Viard, Magnien 2018],...

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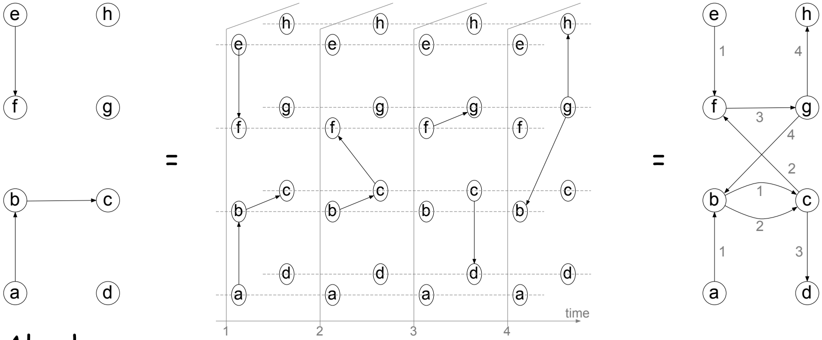

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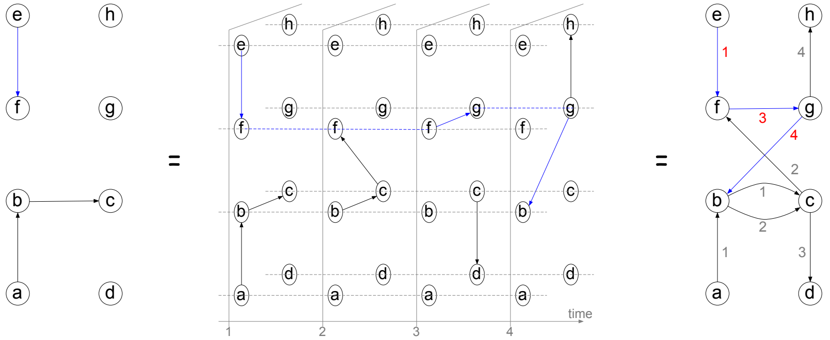
Temporal path/walk



Definitions :

- Simple directed model : a temporal graph is a multi-digraph $D = (V, A)$ with a time labeling $\lambda : A \to \mathbb{N}$ of its edges.

- A path with edges $e_1, \dots, e_k$ is a (strict) temporal path when $\lambda(e_1) < \cdots < \lambda(e_k)$, i.e. time labels (strictly) increase along the path.

Example : $e \xrightarrow{1} f \xrightarrow{3} g \xrightarrow{4} b$ (waiting at f is allowed, $\neg a \longrightarrow f$)

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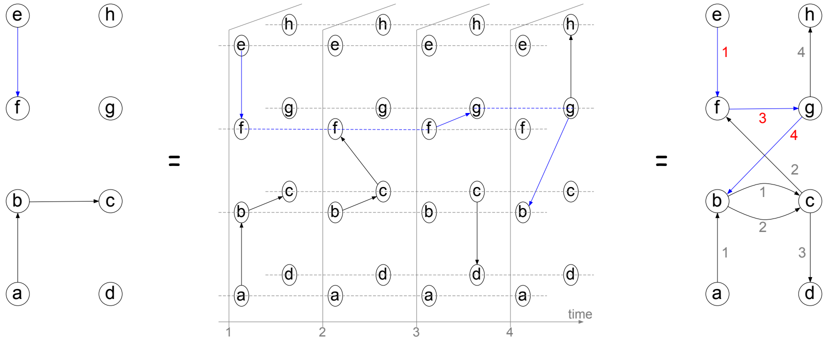

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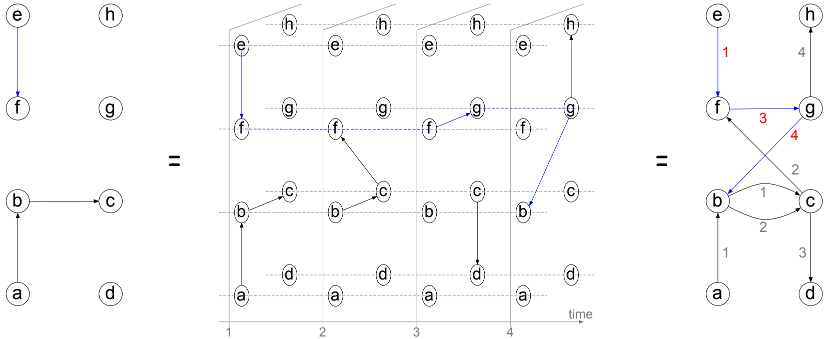

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Temporalization problem

Directed graph temporalization

Problem : given a multi-digraph, assign time labels to edges so as to maximise the number of pairs temporally conneceted.

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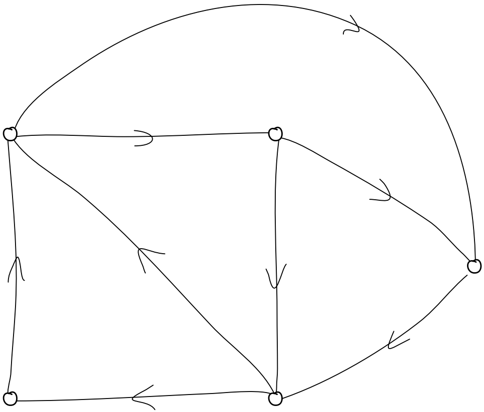

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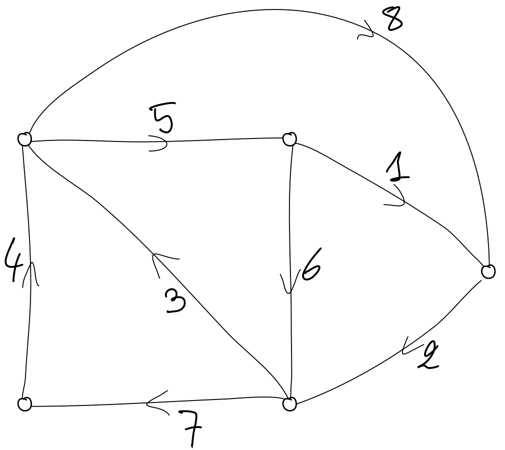

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Original motivation : optimize the schedule of a public transit network.

Also a natural fundamental problem.

The problem is mostly interesting when most pairs can be connected, we thus focus on strong digraphs.

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Related work

Undirected graph version (symmetric digraph, an arc and its reverse must have same time label) : deciding “label connectivity” is NP-complete [Göbel, Cerdeira, Veldman 1991].

Approximation is easy : use a spanning tree and a centroid node...

Minimizing $|\lambda|$, i.e. the number of labels, for achieving temporal connectivity is NP-hard [Klobas, Mertzios, Molter, Spirakis 2022].

It is NP-hard to decide if a strong digraph has a pair of edge-disjoint spanning in-tree and out-tree [Bang-Jensen 1991].

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Given a strong digraph D, deciding whether there exists an assignment of one time label per arc such that all pairs are temporally connected is NP-complete. [Balliu, Brunelli, Crescenzi, Olivetti, V. 2023]

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Problem variants

Problem 1 : Given a strong digraph, compute a temporalization connecting a constant fraction of pairs through strict temporal paths.

Problem 1bis edge-ordering (equivalent) : Compute an arc ordering σ for σ-respecting paths.

Problem node-ordering : a node ordering π for π-forward paths.

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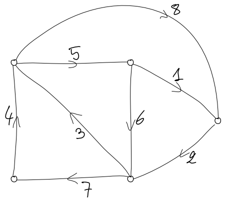

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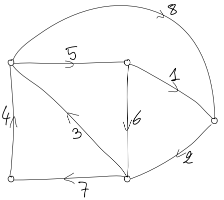

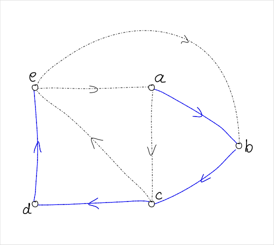

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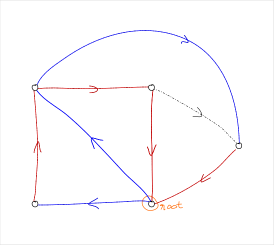
Compute a pair of edge-disjoint in-tree and out-tree both spanning a constant fraction of nodes.



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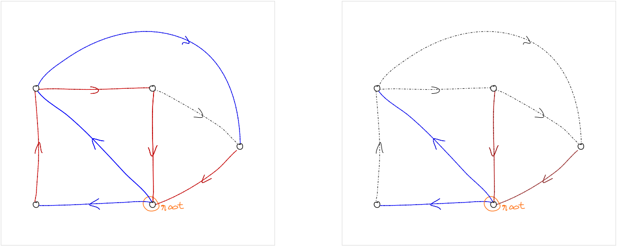

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Main result : approximation

Any strong digraph D has a pair of node-disjtoint in-tree and out-tree both spanning $n/6$ nodes that can be computed in $O(n^2)$ time [Bessy, Thomassé, V. 2023].

Lemma : any strong digraph (V, A) has a balanced cyclic separator C, that is V can be partionned in I, C, O such that :

- C is spanned by a directed cycle,
- there are no arcs from I to O (directed separator),
- both $I \cup C$ and $I \cup O$ has size at least $n/3$ (balanced).

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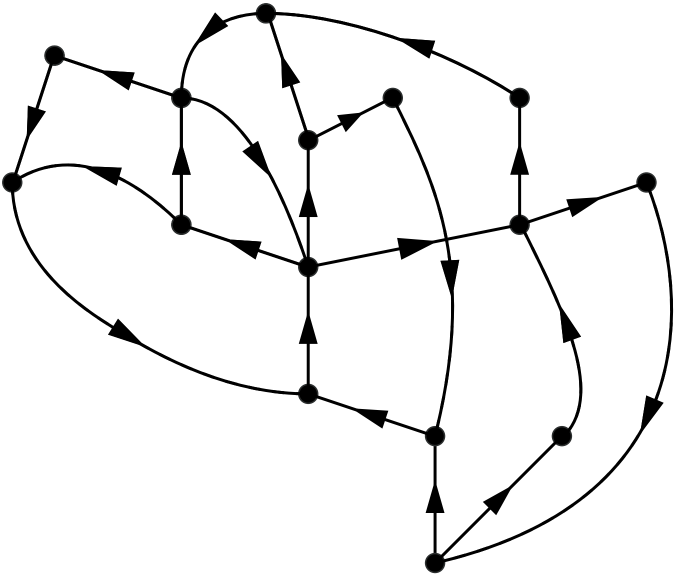

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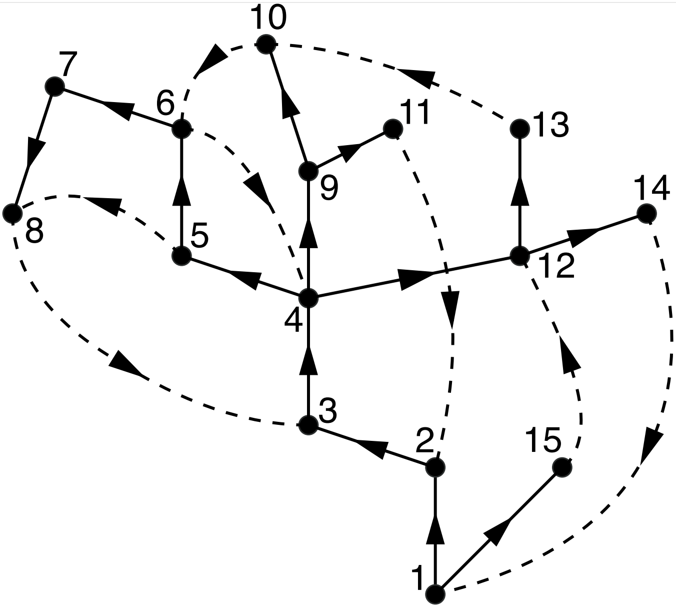

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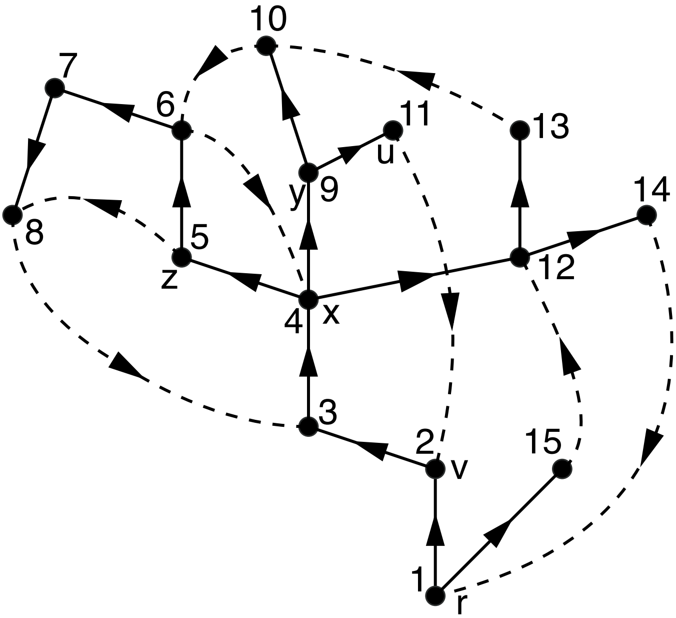

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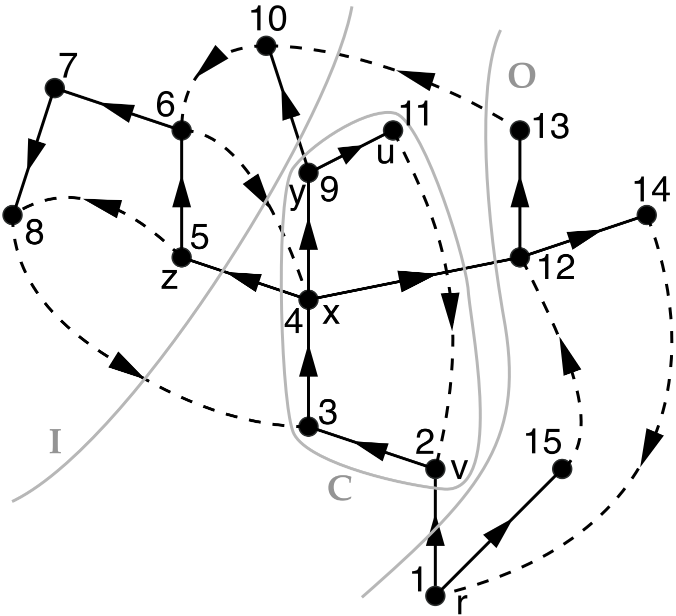

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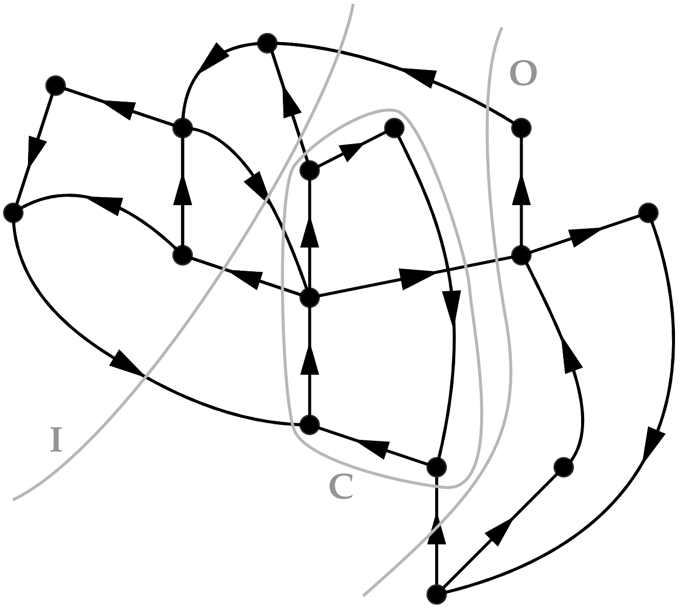

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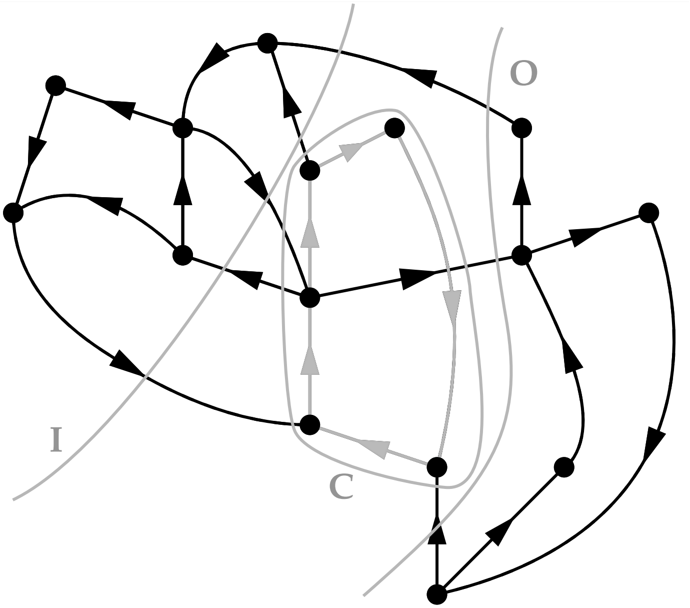

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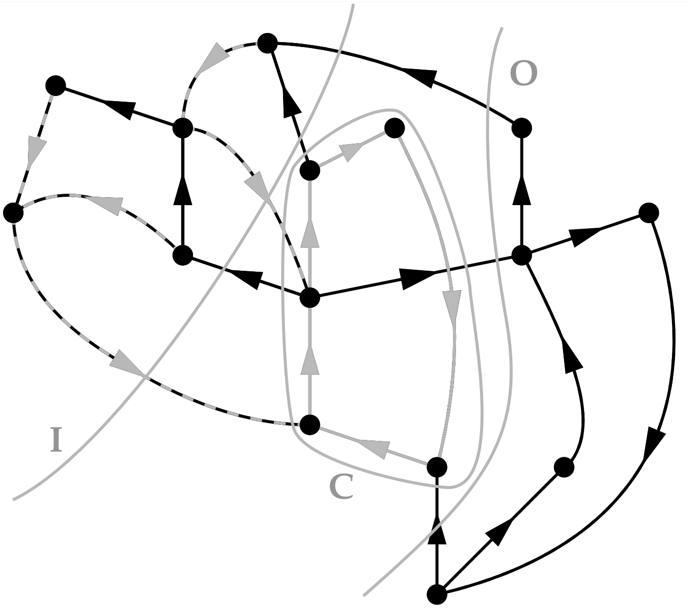

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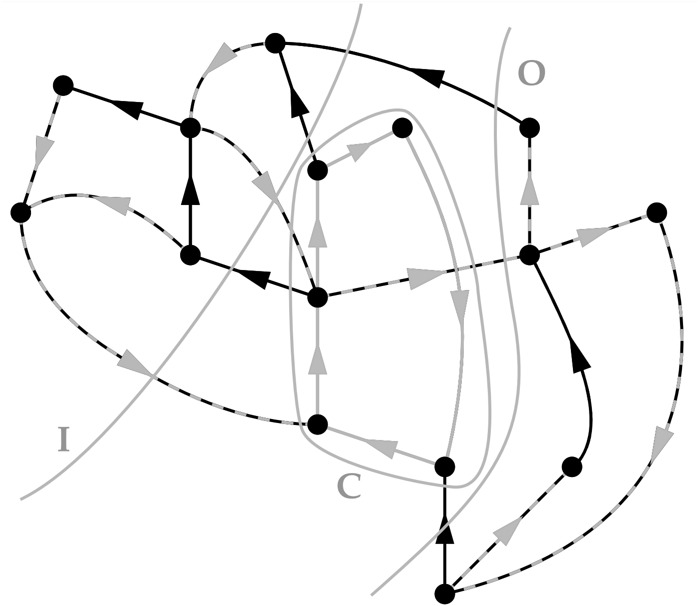

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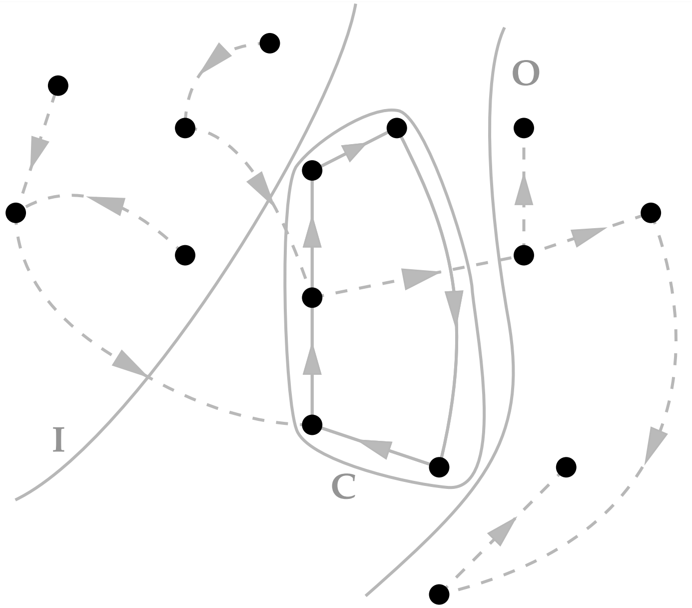

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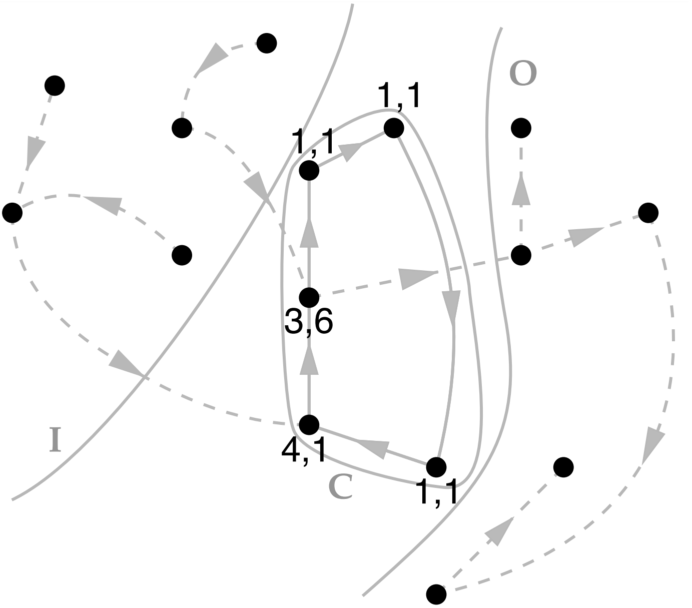

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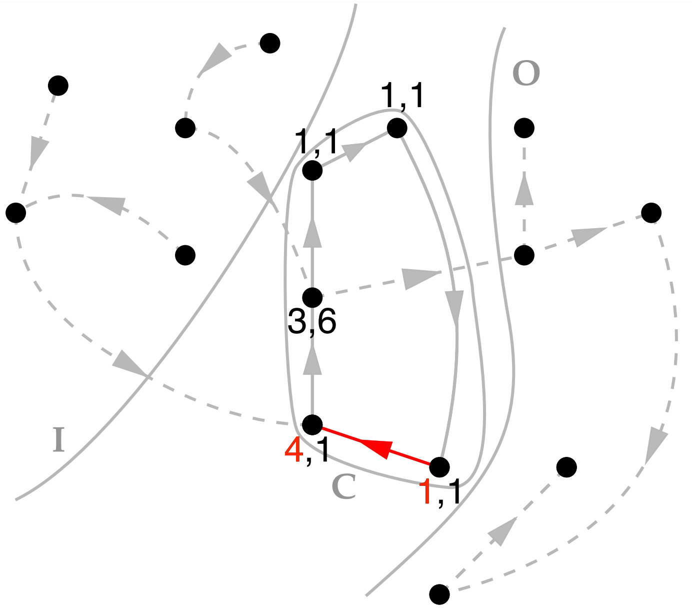

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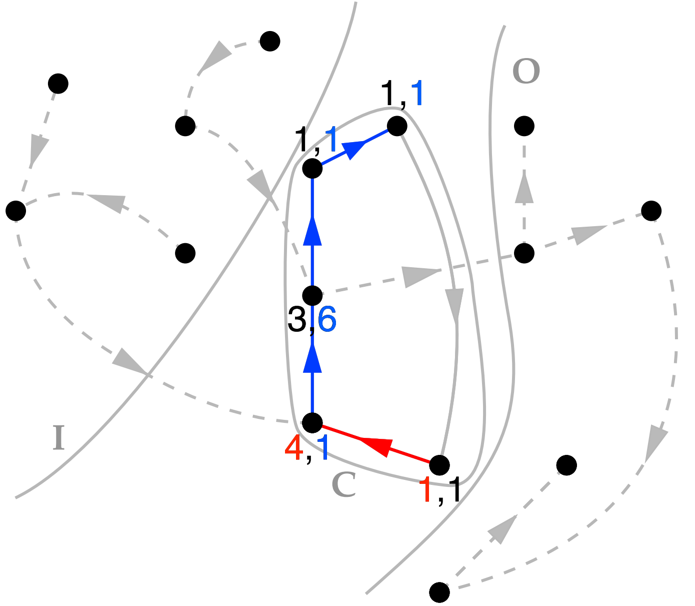

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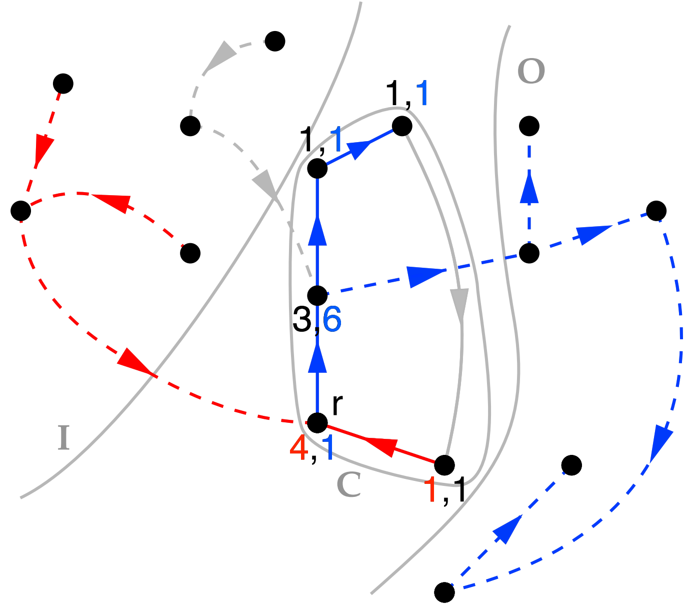

Extensions

The bitree construction generalizes to node weights.

And thus to a subset of nodes U (only pairs in $\binom{U}{2}$ are counted).

But not to an arbitrary set $R \subset \binom{V}{2}$ of requested pairs.

Conjecture : every strong digraph has a $O(\log n)$ forward cover, i.e. $k = O(\log n)$ node orderings such that any pair $\{x, y\}$ is connected by a path respecting one of the k orderings.

Transit networks : trip temporalization [Brunelli, Crescenzi, V. 2023].

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Perspectives

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Is there a gap between edge ordering and node ordering? ($(n/3)^2$ vs $(n/6)^2$?)

What is the complexity of left-maximal DFS?

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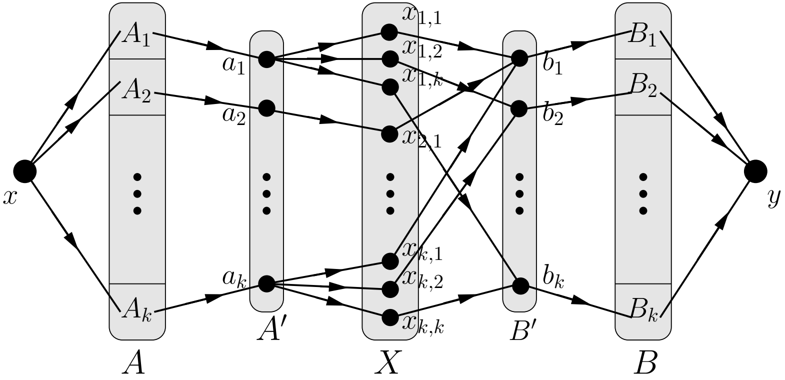
Thanks.

A good node ordering may give poor bitrees.



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Related problem : Find an in-tree and an out-tree with same root that are edge disjoint and have size $\Omega(n)$.

It is NP-hard to decide if a strong digraph has such a pair. [Bang-Jensen 1991]

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Question : How many pairs of in-tree, out-tree are needed to cover all pairs ?

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Given an undirected graph G, deciding whether there exists an assignment of one time label per edge such that all pairs are temporally connected is NP-complete. [Göbel, Cerdeira, Veldman 1991]

Approximation is obvious : take a spanning tree and assign time labels that connect $(n/2)^2$ pairs.

Related (Gossip/telephone problem [Bumby 1981]) : The minimum number of time labels allowing to temporally connect all pairs at least $2n-4$, and equals $2n-4$ if G has a C_4 (one or two time labels per edge).

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Related (Gossip/telephone problem [Bumby 1981]) : The minimum number of time labels allowing to temporally connect all pairs at least $2n-4$, and equals $2n-4$ if G has a C_4 (one or two time labels per edge).

Temporal graph models

Main models (interval availability)

Time-dependent network : a graph where edge delay depends on time [Cooke, Halsey 1966] :
$\mathcal{G} = ((V, E), \delta)$ with $\delta : E \to \mathbb{R}^{\mathbb{R}}$ ($\delta(e)(\tau)$ is the delay of $e \in E$ at time τ).

Time-varying graph : edge and nodes are available at certain times.

Pice-wise constant-delay : each $\delta(e)$ is piecewise constant.
[Bui-Xuan, Ferreira, Jarry 2003], [Dehne, Omran, Sack 2012]

Pice-wise zero-delay : $\delta(e)$ has value 0 or ∞.
Temporal networks [Holme 2015], link-stream [Latapy, Viard, Magnien 2018].

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Main models (point availability)

Evolving graph : sequence of (static) graphs with same vertices $\mathcal{G} = (\mathsf{V}, \mathsf{E}_1, \ldots, \mathsf{E}_\ell)$ [Bhadra, Ferreira 2003] or equivalently time labeled graphs $\mathcal{G} = ((\mathsf{E}, \mathsf{V}), \lambda)$ with $\lambda : \mathsf{E} \to 2^{\mathbb{N}}$ [Michail 2016].
(Delay of edges is 1/0 for strict/non-strict temporal paths.)

Simple : $\mathcal{G} = ((\mathsf{E}, \mathsf{V}), \lambda)$ with $\lambda : \mathsf{E} \to \mathbb{N}$ [Kempe, Kleinberg, Kumar 2002].
Proper : edges incident to a node have pairwise disjoint labels (strict and non-strict then coincide) [Casteigts, Corsini, Sarkar 2022].
Happy : simple and proper [Casteigts, Corsini, Sarkar 2022].
Globally happy : simple and pairwise-disjoint time labels.

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What is time?

Time domain : $\mathbb{N}$ or $\mathbb{R}$ or $\mathbb{T}$?

Discrete :

- time is discrete,
- and/or edges are available at some given points in time,
- and/or traversal takes time 1 (or 0).

Continous :

- time is continuous,
- and/or edges are available during given intervals of time,
- and/or traversal takes constant time.

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