Realizing Temporal Transportation Trees

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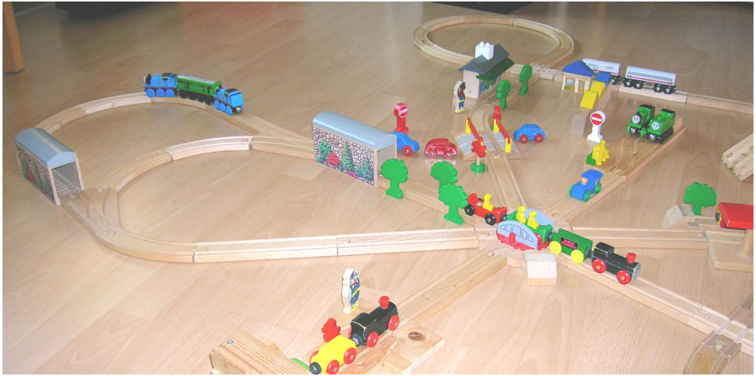
Algorithmic Aspects of Temporal Graphs VII

Problem Setting

Imagine you are given a train track system.



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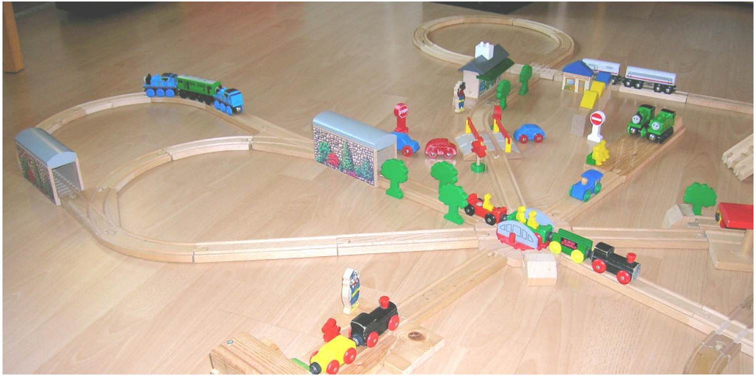

Your task is to design a (periodic) schedule for the trains, such that travel times are sufficiently fast.

Restriction: Transportation Trees

Restriction: Train track system has a tree structure.

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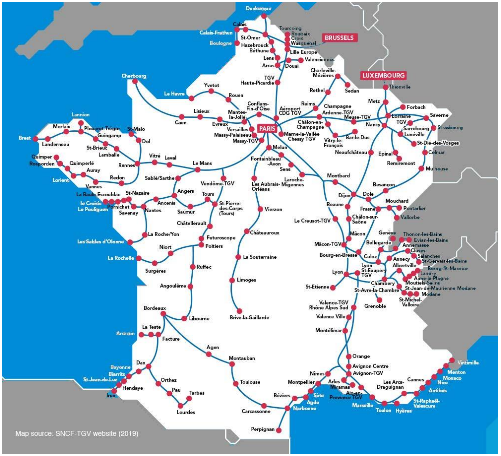

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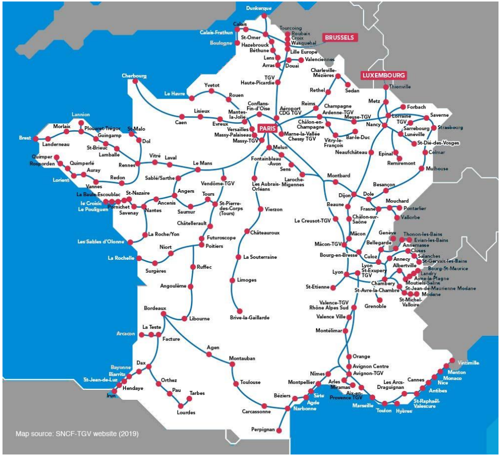

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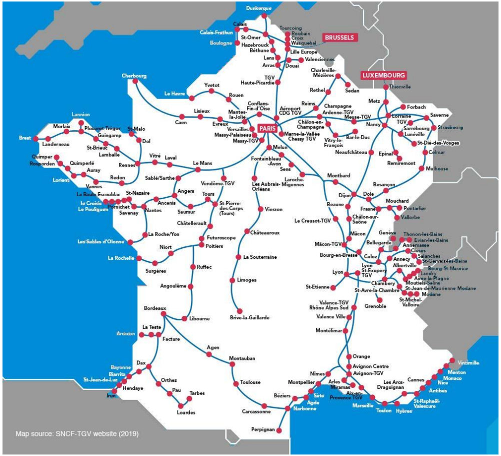

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Labels define **periodic temporal graph** $\mathcal{G} = (V, E, \lambda)$ where for all $e \in E$:

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Travel time $\delta(u, v)$ from u to v is **duration** of **fastest** temporal path from u to v in $\mathcal{G}$.

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Duration of a fastest path: Last label $-$ first label $+$ one.

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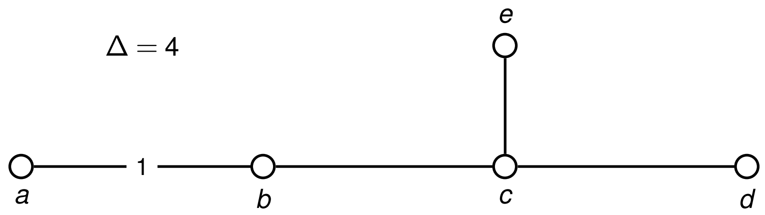

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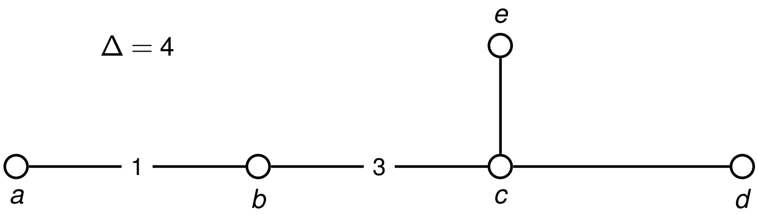

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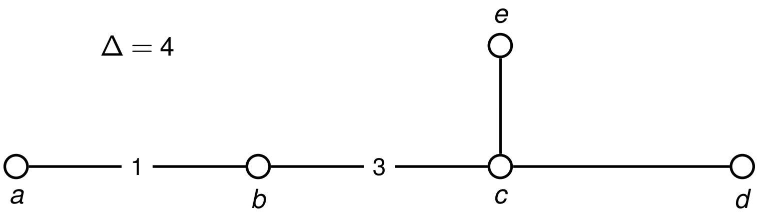

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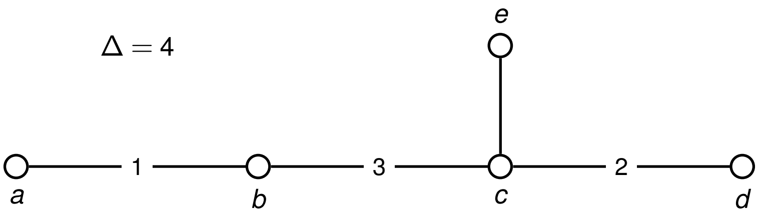

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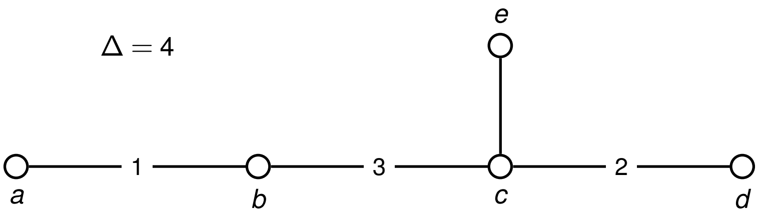

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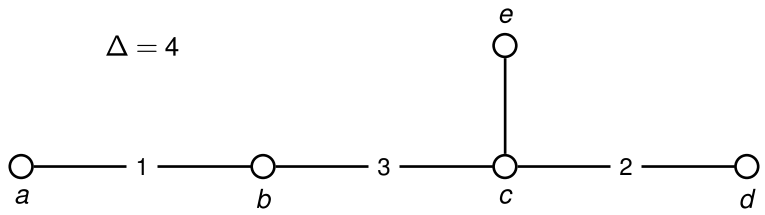

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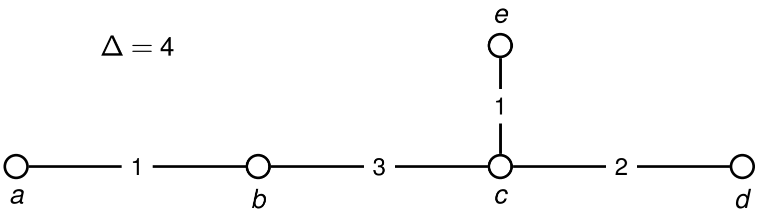

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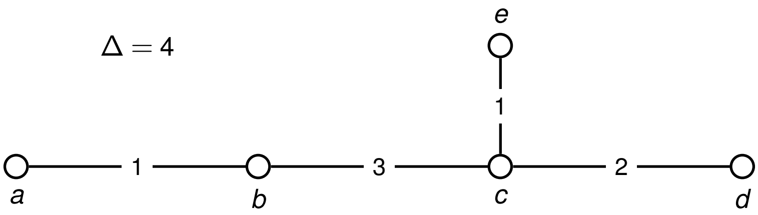

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TTR is **fixed-parameter tractable** w.r.t. the **number of leaves of the tree.**

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- Further results.

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Input: A vector x of n variables of which some are considered integer variables, a constraint matrix $A \in \mathbb{R}^{m \times n}$, two vectors $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, and a target value $t \in \mathbb{R}$.

Question: Is there an assignment to the variables such that all integer variables are set to integer values, $c^\intercal x \geq t$, $Ax \leq b$, and $x \geq 0$?

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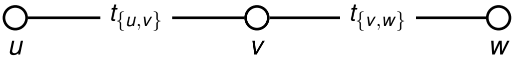
Lemma

If the constraint matrix for the fractional variables is **totally unimodular**, then the MILP admits an optimal solution where **all variables are set to integer values**.

Travel Delays

Travel Delays: Let v be a vertex and u, w be neighbors of v. Let $t_{\{u,v\}}$ denote the label on edge $\{u, v\}$ and let $t_{\{v,w\}}$ denote the label on edge $\{v, w\}$.



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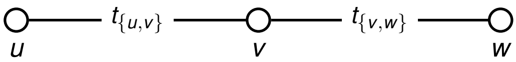

Then the **travel delay** $\tau_v^{u,w}$ from u to w at vertex v is defined as follows.

$$\tau_v^{u,w} = \begin{cases} t_{\{v,w\}} - t_{\{u,v\}}, & \text{if } t_{\{v,w\}} > t_{\{u,v\}}, \\ t_{\{v,w\}} - t_{\{u,v\}} + \Delta, & \text{otherwise.} \end{cases}$$

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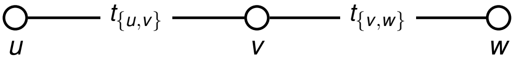

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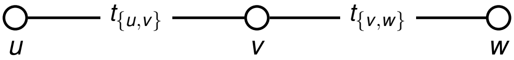

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Labels computable from travel delays!

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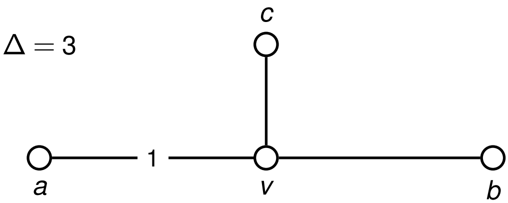

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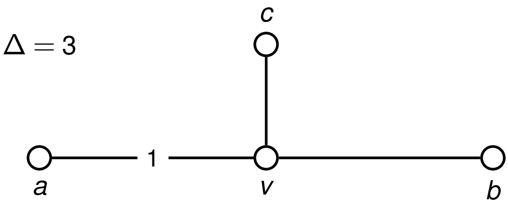

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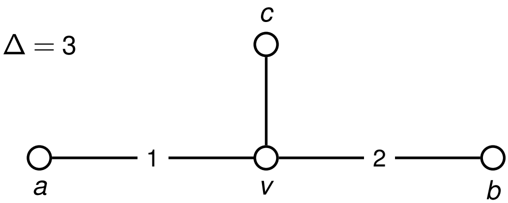

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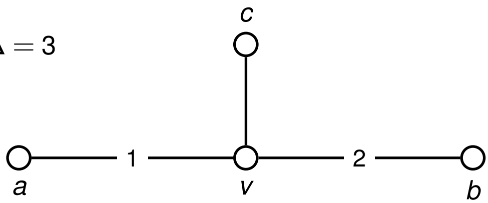

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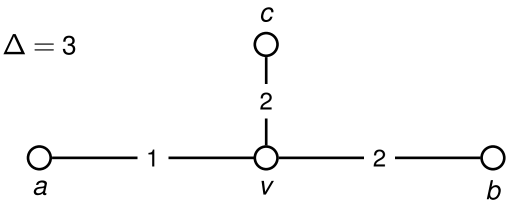

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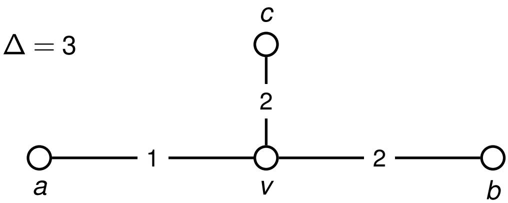

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Travel delays not realizable!

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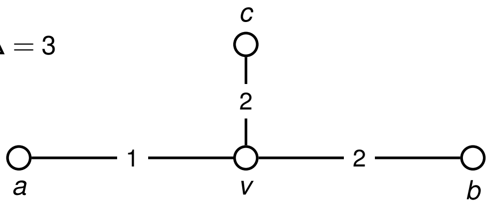

MILP Formulation I

Main Idea: Create a **fractional variable** for each travel delay.
Add constraint for each vertex pair s, t checking that $\delta(s,t) \leq D(s,t)$.

Problem: High degree vertices (degree larger than two).

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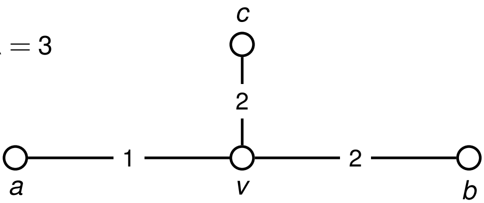

Observation

Let G be a tree with ℓ leaves. Then the following holds.

- The maximum degree of G is at most ℓ.
- The number of vertices with degree larger than two is at most ℓ.

MILP Formulation II

Main Idea:

- Create a **fractional variable** $x_v^{a,b}$ for each travel delay from a to b at vertex v.
  Assume vertices are ordered and $a < b$.

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Duration Constraints: For each pair of vertices s, t we have the following constraint.

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MILP Formulation II

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MILP Formulation III

Formal construction of the directed tree:

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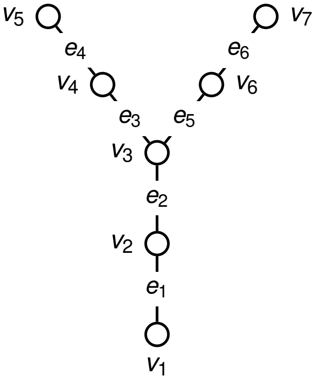

Tree G.

MILP Formulation III

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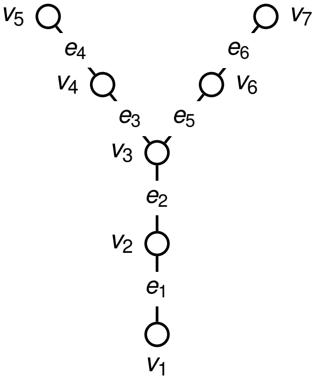

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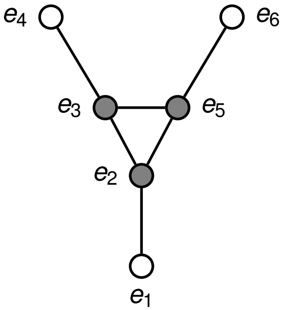

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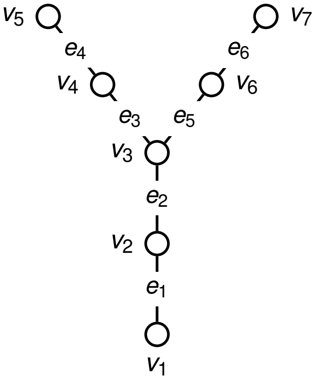

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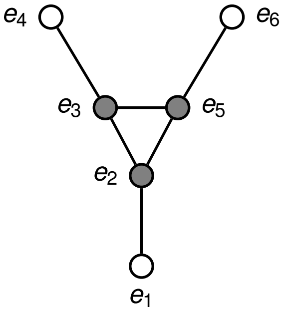

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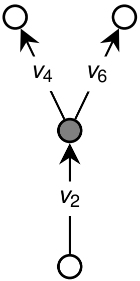

Directed tree T.

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Thank you!