Parameterized Algorithms for Multi-Label Periodic Temporal Graph Realization

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(slides (mostly) by Nils Morawietz)

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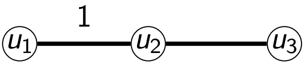

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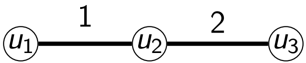

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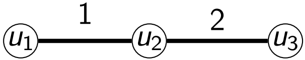

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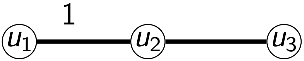

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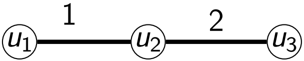

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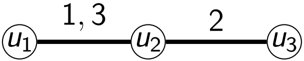

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For each $\ell \geq 5$, *Multi-Label Periodic TGR is NP-hard on stars.*

Ideas of the reduction for hardness on stars

Reduction from **Vertex Cover** ("is there a **vertex cover** of size k in the graph G?")

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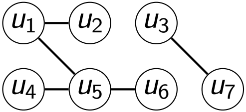

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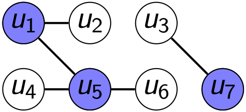

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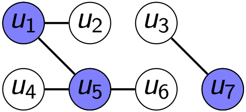

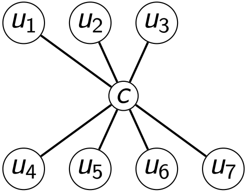

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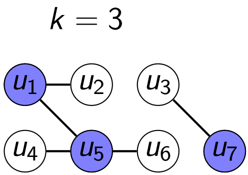

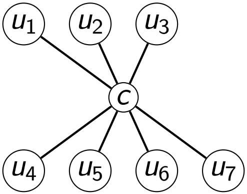

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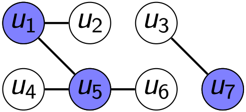

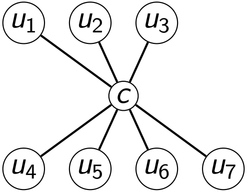

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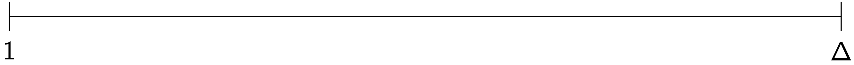

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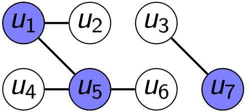

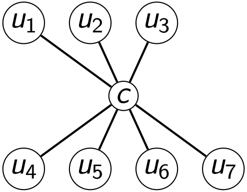

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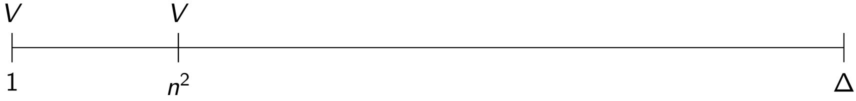

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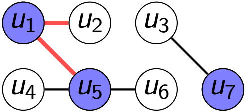

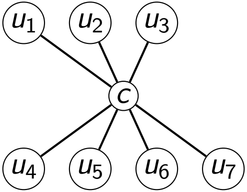

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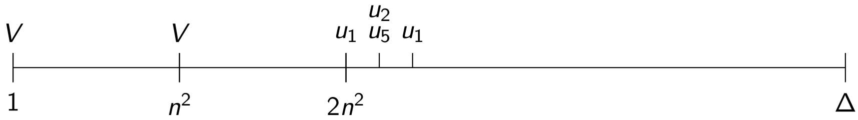

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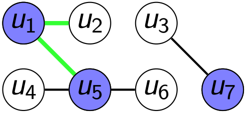

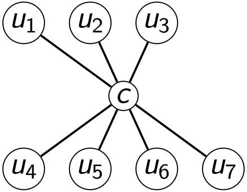

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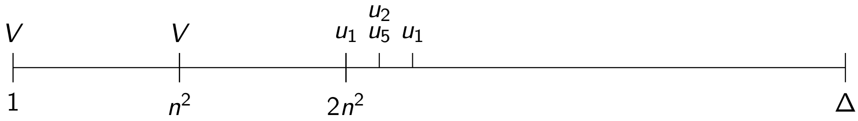

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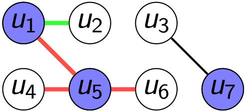

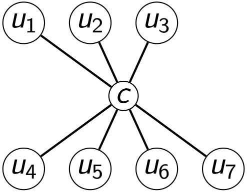

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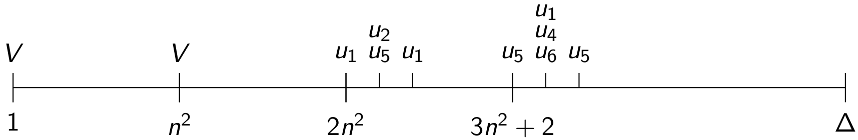

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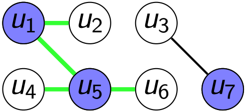

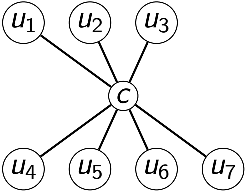

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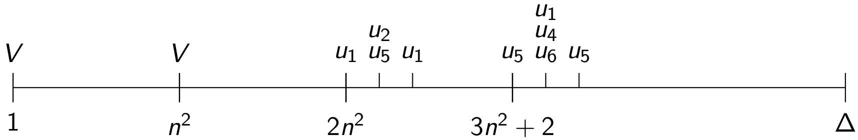

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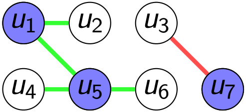

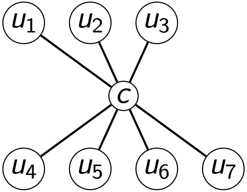

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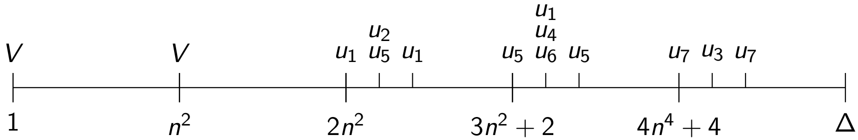

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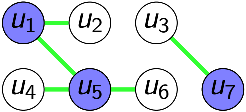

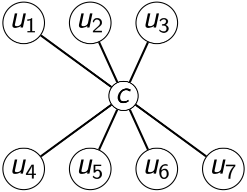

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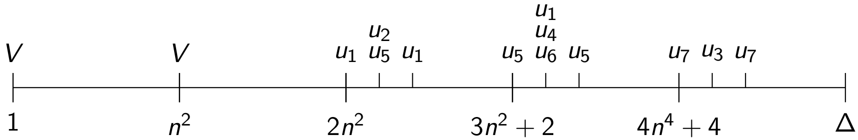

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$\mathrm{vc} \mathrel{\hat{=}}$ vertex cover number, $\mathrm{nu} \mathrel{\hat{=}}$ number of non-universal vertices

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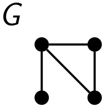

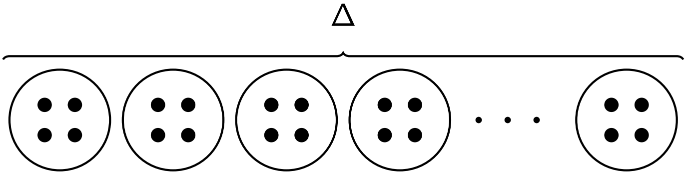

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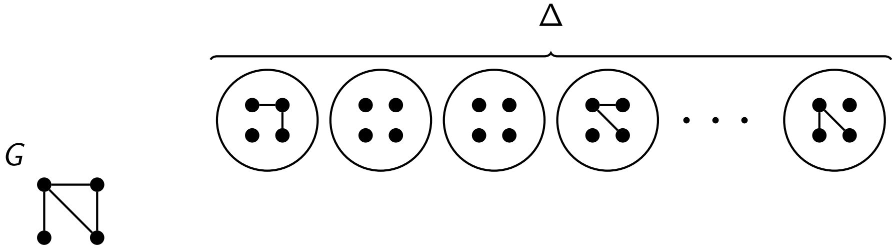

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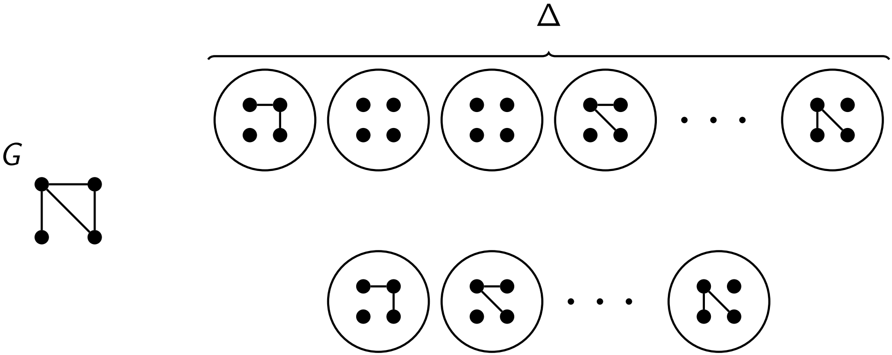

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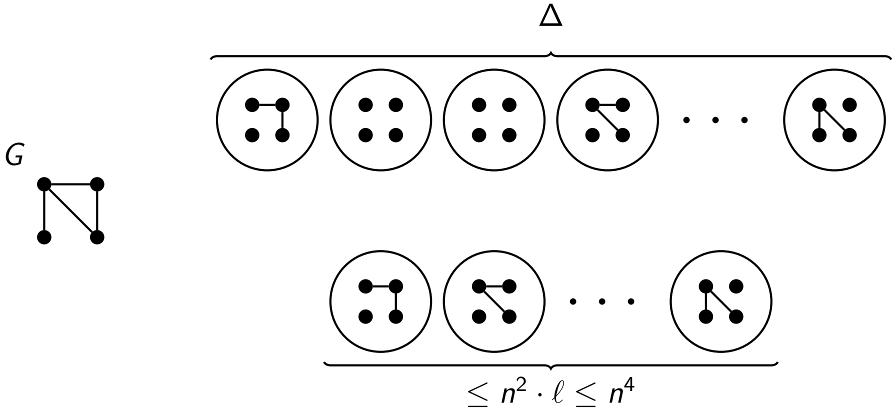

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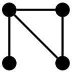
G



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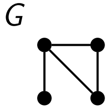

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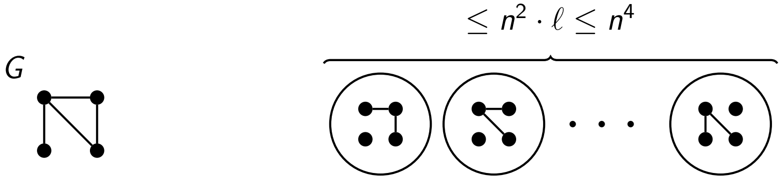

Guess the $\mathcal{O}(n^4)$ non-empty snapshots (in $n^{\mathcal{O}(n^4)}$ time)

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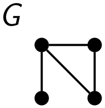

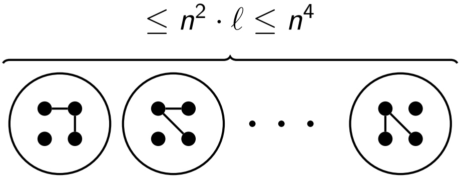

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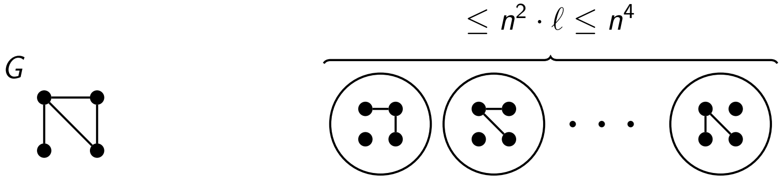

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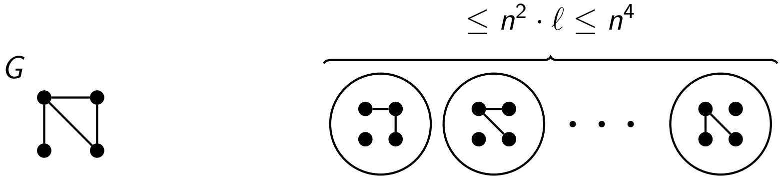

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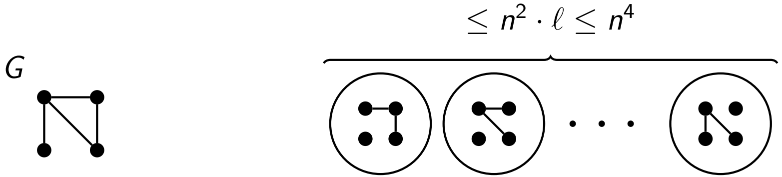

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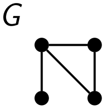

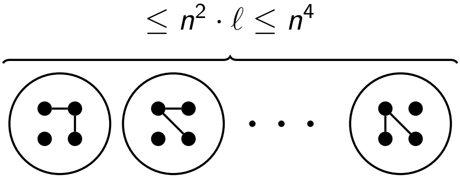

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Theorem

Multi-Label Periodic TGR can be solved in $n^{\mathcal{O}(n^4)}$ time.

Details of the ILP

Determine whether a set L of non-empty snapshots can be assigned to time steps so that the temporal graph realizes D:

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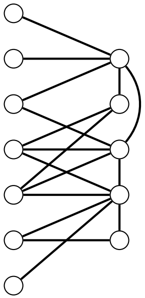
ILP:

$$
\begin{aligned}
t_j + z\Delta - t_i + 1 &= D_{uv}, && \forall (u, v), i = s_{uv}, a(u, v, i) = (j, z) \\
t_j + z\Delta - t_i + 1 &\geq D_{uv}, && \forall (u, v), i \neq s_{uv}, a(u, v, i) = (j, z) \\
t_1 &\geq 1 \\
t_i - t_{i-1} &\geq 1, && \forall i : 2 \leq i \leq L \\
t_L &\leq \Delta
\end{aligned}
$$

Parameter combination vc + Δ



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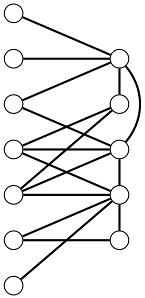

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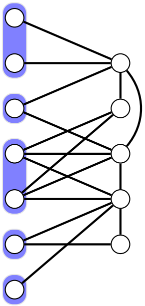

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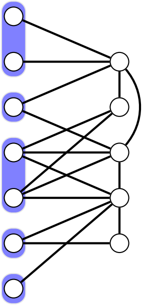

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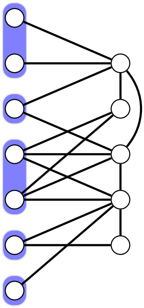

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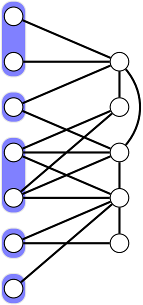

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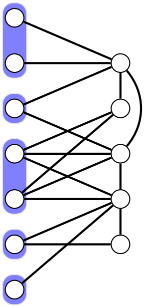

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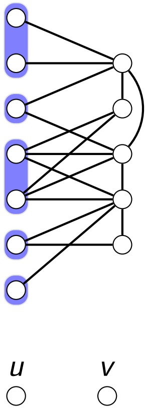

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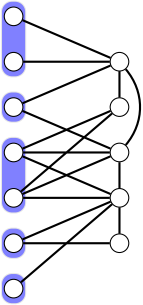

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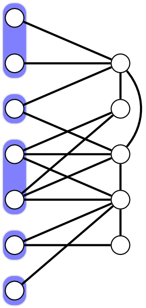

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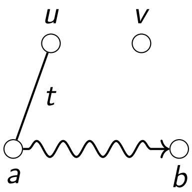

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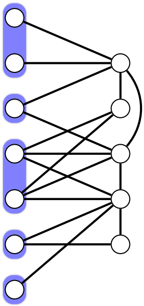

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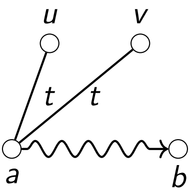

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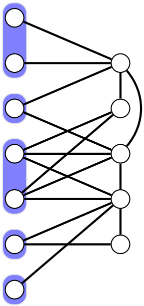

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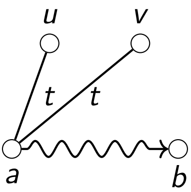

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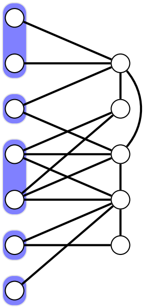

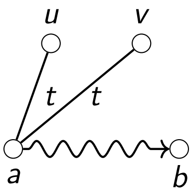

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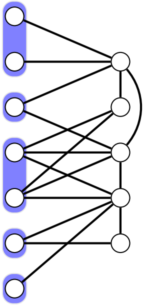

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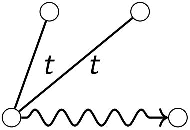

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- Consider non-strict instead of strict temporal paths.

Thank you!

Questions?