Temporal Network Creation Games

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Algorithmic Aspects of Temporal Graphs VI

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Temporal Network Creation Games

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TEMPORAL REACHABILITY PROBLEM

[Kempe, Kleinberg, and Kumar, JCSS 2002]

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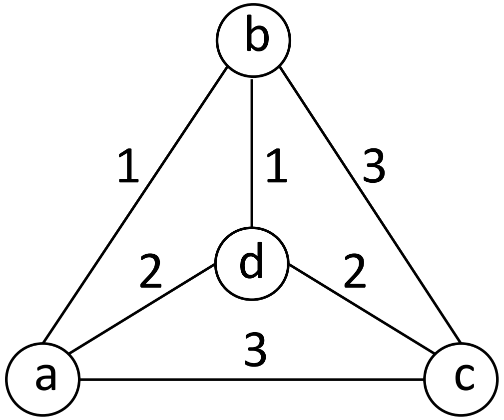
NETWORK CREATION GAMES

First paper in AGT [Fabrikant, Luthra, Maneva, Papadimitriou, Shenker, PODC 2003]

Afterwards, many other models were defined and studied

Temporal Graphs



A temporal graph
with lifetime $t = 3$

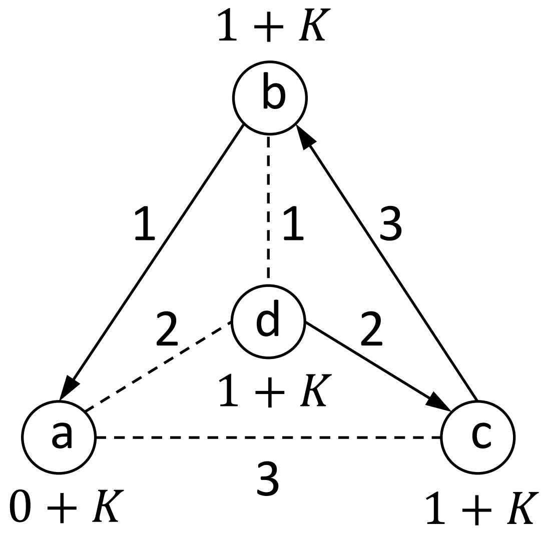
Temporal graph H with lifetime $t \in \mathbb{N}$:

1. undirected graph with n vertices
2. each edge e has an integer time label $\lambda(e) \in [t]$

(non-strict) temporal path from v to u in H: A path from v to u with monotonically non-decreasing time labels (u is **temporally reachable** from v)

Temporally connected graph: A temporal graph H that contains a temporal path from v to u, for every ordered pair (v, u) of vertices v, u of H

Temporal Reachability Network Creation Game



- *Complete* temporal host graph H with lifetime t
- vertices of H are *selfish* agents

HOW THE AGENTS FORM A NETWORK

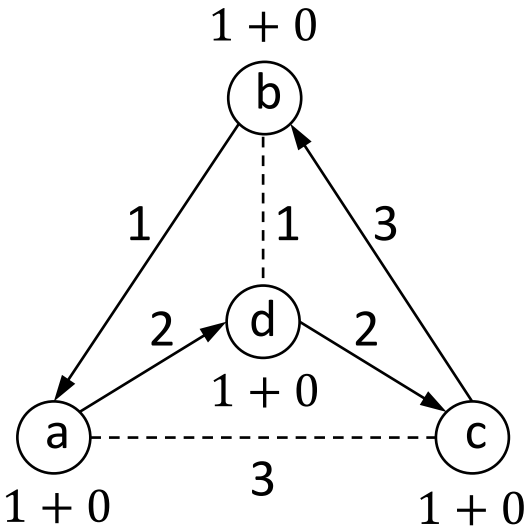
- agent v buys a set of edges S_v incident to her
- Let $S = \langle S_v \rangle_v$ be the strategy profile
- $\bigcup_v S_v$ are the edges of the formed network $G(S)$

THE COST FUNCTION EACH AGENT WANTS TO MINIMIZE

- U_v: set of vertices that are NOT temporally reachable from v in $G(S)$
- penalty of $K > 1$ for each unreached vertex

$$cost(v, S) = |S_v| + K \cdot |U_v|$$

Temporal Reachability Network Creation Game



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$$cost(v, S) = |S_v| + K \cdot |U_v|$$

$G(S)$ is a ***stable*** if $cost(v, S) \leq cost(v, (S_{-v}, S'_v))$ for every agent v and for every alternative strategy S'_v for agent v.

Important questions in NCGs

- Analyzing the EXISTENCE of STABLE NETWORKS
  - **QUESTION 1:** Does the game always admit stable networks?
- Understanding the COMPUTATIONAL COMPLEXITY aspects of the game
  - **QUESTION 2:** Is agent x playing a best response?
  - **QUESTION 3:** Is the network formed by the agents stable?
- Analyzing the QUALITY of STABLE NETWORKS
  - **QUESTION 4:** What is the *Price of Anarchy* of the game?

Price of Anarchy: worst-case ratio between social cost of stable networks vs social cost of optimum solution, where social cost of S is equal to $\sum_v cost(v, S)$

Trivial results (lifetime $t = 1$)

QUESTION 1: Does the game always admit stable networks?	YES
QUESTION 2: Is agent x playing a best response?	in P
QUESTION 3: Is the network formed by the agents stable?	in P
QUESTION 4: What is the Price of Anarchy of the game?	1 (spanning trees)

Our results (for lifetime $t \geq 2$)

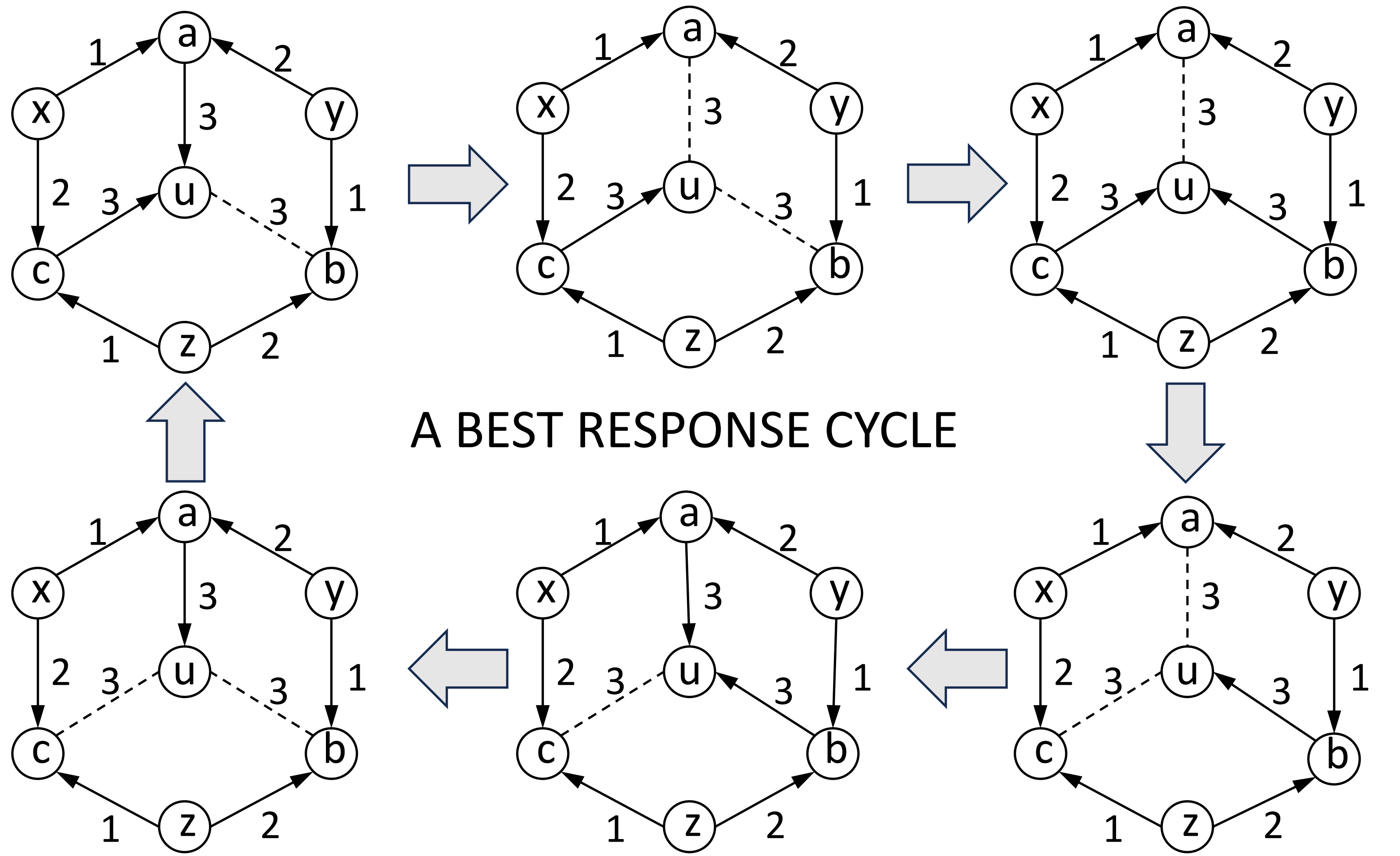
QUESTION 1: Does the game always admit stable networks?	
QUESTION 2: Is agent x playing a best response?	
QUESTION 3: Is the network formed by the agents stable?	
QUESTION 4: What is the Price of Anarchy of the game?	

EXISTENCE of STABLE NETWORKS

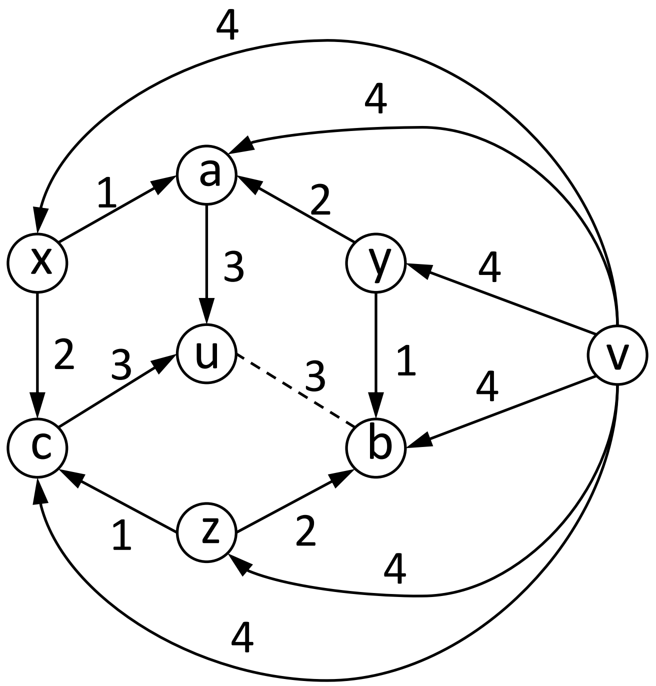
- **QUESTION 1:** Does the game always admit stable networks?
- **FIRST ATTEMPT:** Is the game a *potential game*?

QUESTION 1: Does the game always admit stable networks?

THEOREM: The Temporal Reachability NCG is NOT a potential game.



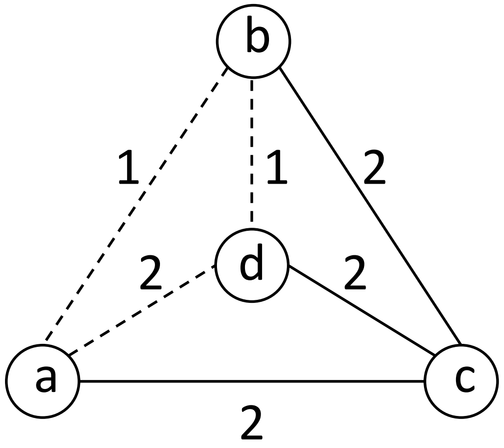
Detail to guarantee temporal connectivity



QUESTION 1: Does the game always admit stable networks?

THEOREM: Complete host graphs with lifetime $t = 2$ always contain a stable network.

PROOF: There is always a spanning tree whose edges all have the same time label.



Our results (for lifetime $t \geq 2$)

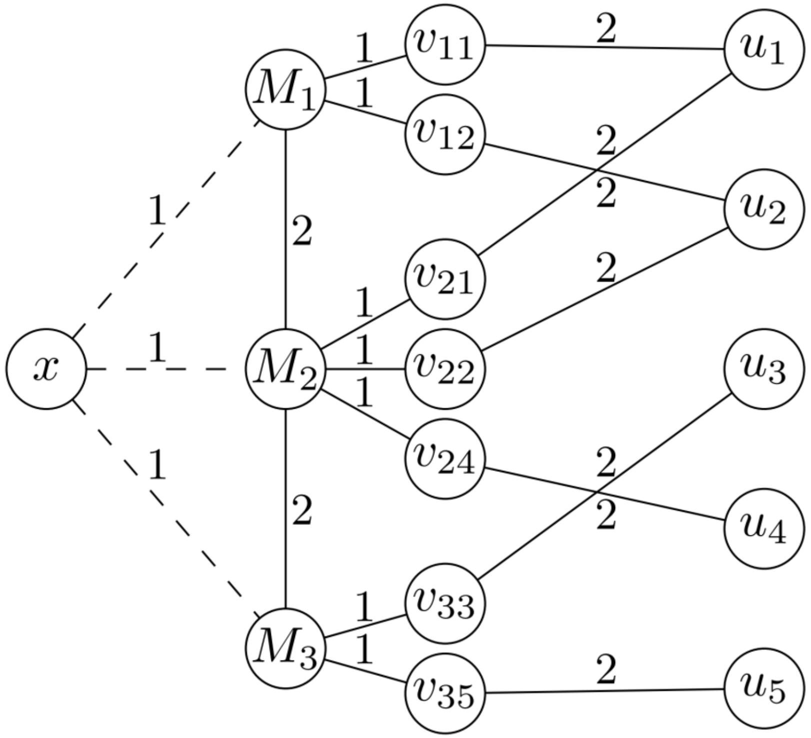
QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	
QUESTION 3: Is the network formed by the agents stable?	
QUESTION 4: What is the Price of Anarchy of the game?	

COMPUTATIONAL COMPLEXITY

- **QUESTION 2:** Is agent x playing a best response?
- **QUESTION 3:** Is the network formed by the agents stable?

QUESTION 2: Is agent x playing a best response?

THEOREM: Computing a best response of agent x is NP-hard even for complete host graphs with lifetime $t = 2$.



Missing edges all have time label 2

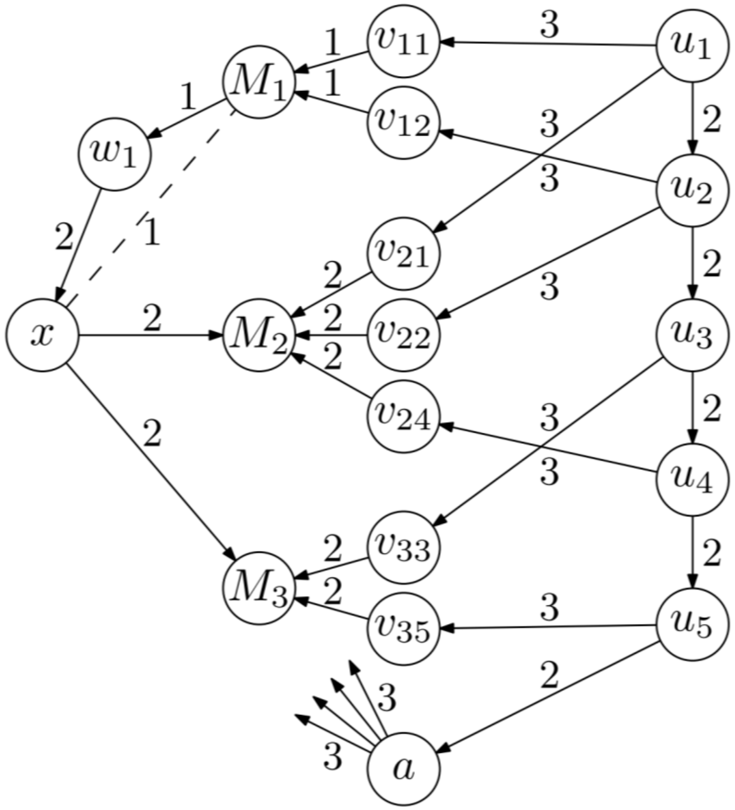
SET COVER PROBLEM INSTANCE:

- Universe $U = \{u_1, u_2, u_3, u_4, u_5\}$
- Sets: $M_1 = \{u_1, u_2\}, M_2 = \{u_1, u_2, u_4\}, M_3 = \{u_3, u_5\}$

Computing a best response for x is equivalent to solving the SET COVER PROBLEM INSTANCE.

QUESTION 3: Is the network formed by the agents stable?

THEOREM: Deciding whether the network formed by the agents is stable is NP-complete even for complete host graphs with lifetime $t = 3$.



Missing edges all have time label 3

Given a SET COVER PROBLEM INSTANCE + SOLUTION, deciding whether there is a better solution is NP-complete.

SET COVER PROBLEM INSTANCE

- Universe $U = \{u_1, u_2, u_3, u_4, u_5\}$
- Sets: $M_1 = \{u_1, u_2\}, M_2 = \{u_1, u_2, u_4\}, M_3 = \{u_3, u_5\}$

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- SOLUTION $\{M_2, M_3\}$

Our results (for lifetime $t \geq 2$)

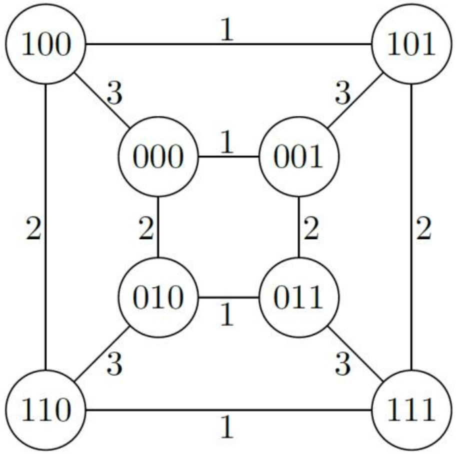
QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	NP-hard
QUESTION 3: Is the network formed by the agents stable?	NP-hard for $t \geq 3$
QUESTION 4: What is the Price of Anarchy of the game?	

QUALITY of STABLE NETWORKS

- **QUESTION 4:** What is the Price of Anarchy of the game?

Our results (for lifetime $t \geq 2$)

QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	NP-hard
QUESTION 3: Is the network formed by the agents stable?	NP-hard for $t \geq 3$
QUESTION 4: What is the Price of Anarchy of the game?	$\Omega(\log n)$



temporally connected 3-d hypercube

Kempe, Kleinberg, and Kumar [JCSS 2002] constructed a temporally connected $\log n$-dimensional hypercube with lifetime $t = \log n$ that is minimal.

- $G(S)$ is the temporal $\log n$-dimensional hypercube
- missing edges all have the same time label of $1 + \log n$

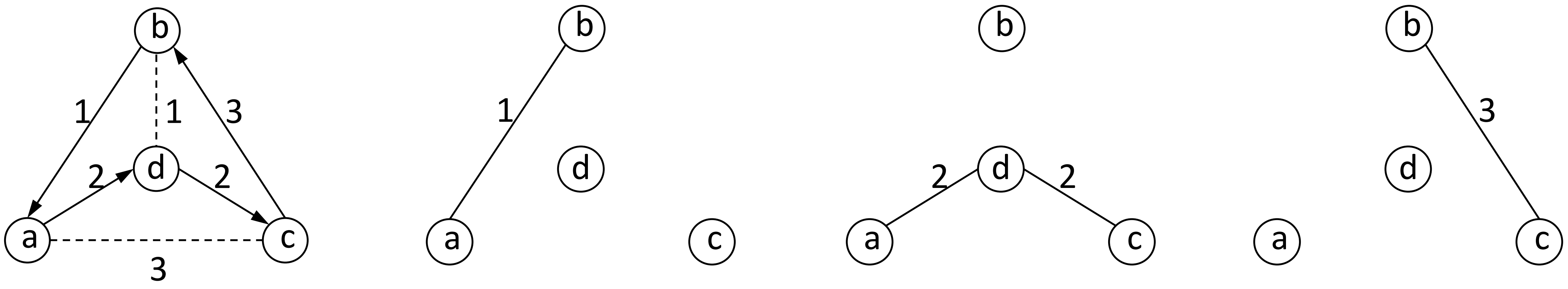
LEMMA: $G(S)$ is stable.

QUESTION 4: What is the Price of Anarchy of the game?

THEOREM: Size of stable networks is $\leq t(n-1)$, where t is the lifetime of host graph.

PROOF: The subgraph G_l of a stable network $G(S)$ formed by edges of time label l is a forest.

A stable network $G(S)$ $\quad G_1 \quad G_2 \quad G_3$



Our results (for lifetime $t \geq 2$)

QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	NP-hard
QUESTION 3: Is the network formed by the agents stable?	NP-hard for $t \geq 3$
QUESTION 4: What is the Price of Anarchy of the game?	$\Omega(\log n)$ $\leq t$

QUESTION 4: What is the Price of Anarchy of the game?

THE CONCEPT OF **NECESSARY EDGE** IN STABLE NETWORKS

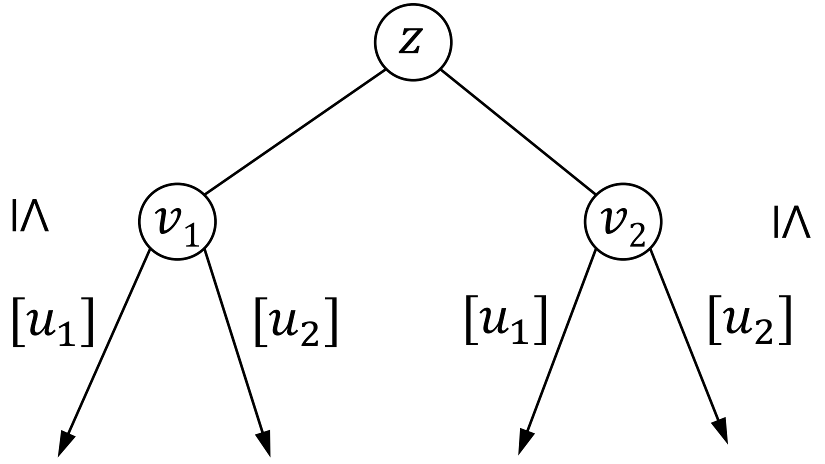
Agent v buys an edge e because e is **necessary** for v to temporally reach some u in $G(S)$.

THEOREM: If $G(S)$ contains $\omega(n\sqrt{n})$ edges, then it is not stable.

PROOF: A counting argument shows that the forbidden structure appears.

Forbidden structure that never appears in stable networks

Label edge e bought by v with $[u]$ if e is necessary for v to reach u



One of the two agents v_1 and v_2 can delete an edge

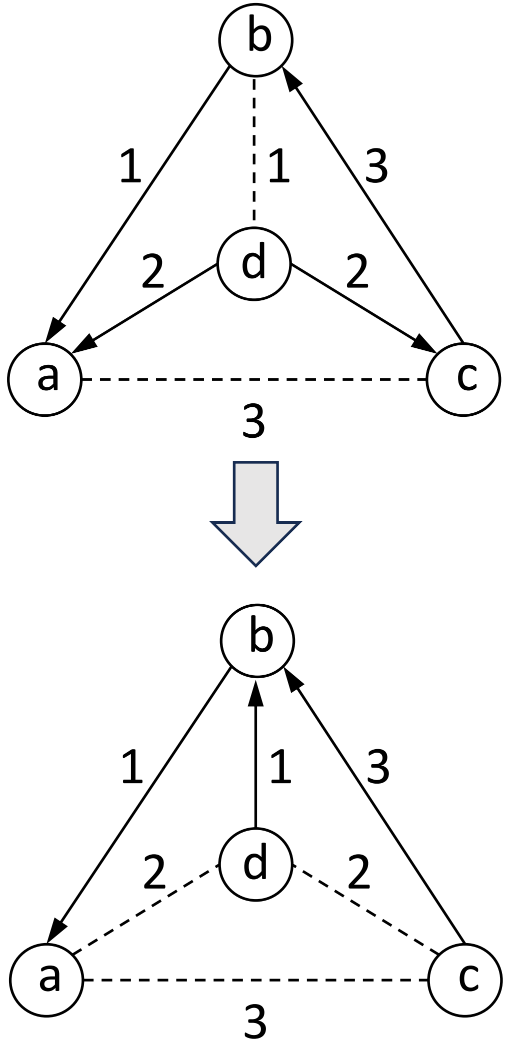
Our results (for lifetime $t \geq 2$)

QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	NP-hard
QUESTION 3: Is the network formed by the agents stable?	NP-hard for $t \geq 3$
QUESTION 4: What is the Price of Anarchy of the game?	$\Omega(\log n)$ $\leq min\{t, O(\sqrt{n})\}$

Extensions to GREEDY agents

The model of GREEDY agent

A greedy stable network which is not stable

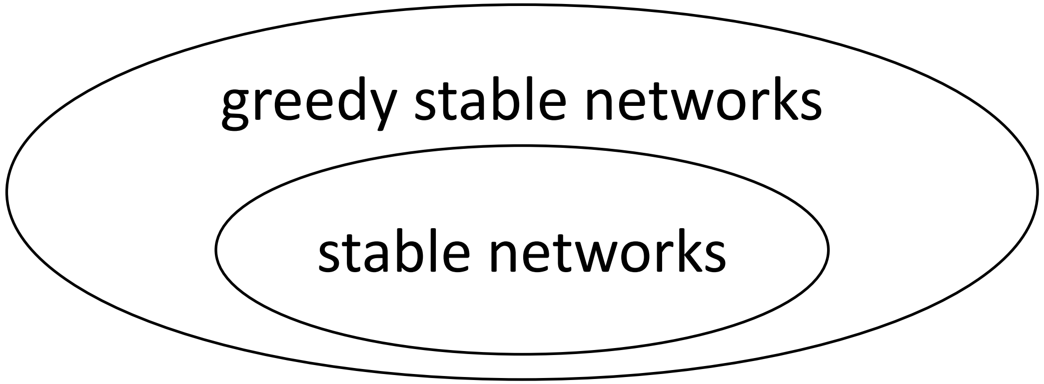


Introduced by [Lenzner, WINE 2012] to avoid hardness of computing best responses.

Strategy allowed to a **greedy** agent v in $G(S)$

- Add an edge incident to v to S_v
- Delete an edge from S_v
- Swap an edge (one deletion + one addition)

$G(S)$ is ***greedy stable*** if no greedy agent can be better off by performing a greedy move.



Our results (for lifetime $t \geq 2$)

	Selfish agents	**Greedy agents**
QUESTION 1: Does the game always admit stable networks?	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game	YES for $t = 2$ OPEN for $t \geq 3$ not a potential game
QUESTION 2: Is agent x playing a best response?	NP-hard	in P
QUESTION 3: Is the network formed by the agents stable?	NP-hard for $t \geq 3$	In P
QUESTION 4: What is the Price of Anarchy of the game?	$\Omega(\log n)$ $\leq min\{t, O(\sqrt{n})\}$	$\Omega(\log n)$ $\leq min\{t, O(\sqrt{n})\}$

Summary and Open Problems

- **MAIN NOVELTY**: We combined Temporal Graphs with Network Creation Games
- We provided some results for Temporal Reachability Network Creation Game

OPEN PROBLEMS

1. **QUESTION 1:** Does the game always admit stable networks?
   - **CONJECTURE:** Yes
2. **QUESTION 4:** What is the Price of Anarchy of the game?
   - **CONJECTURE:** Stable networks are sparse
3. Study other Temporal Network Creation Games

Thanks for your attention