Counting Temporal Paths

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Algorithmic Aspects of Temporal Graphs V

Project initiated at Dagstuhl Seminar "Temporal Graphs: Structure, Algorithms, Applications".

Temporal Graphs and Temporal Paths: Notation and Definition

Temporal Graph

A **temporal graph** $\mathscr{G} = (V, (E_i)_{i \in [T]})$ is a vertex set V with a list of edge sets $E_1, \ldots, E_T$ over V, where T is the lifetime of $\mathscr{G}$.

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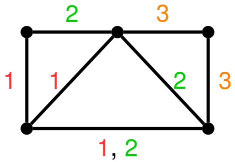

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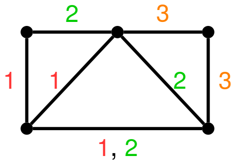
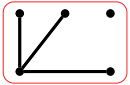


G_1:



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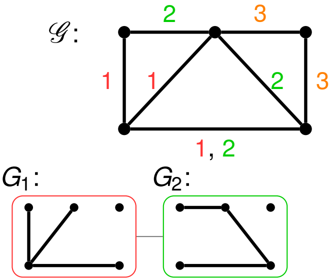

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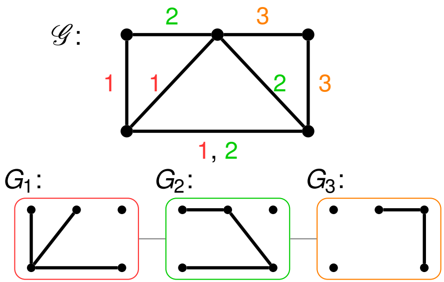

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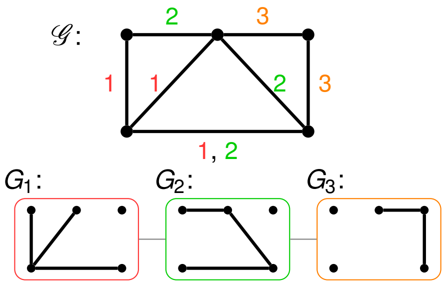

Temporal (s, z)-Path

Sequence of time edges forming a path from s to z that have:

- increasing time stamps (strict).
- non-decreasing time stamps (non-strict).

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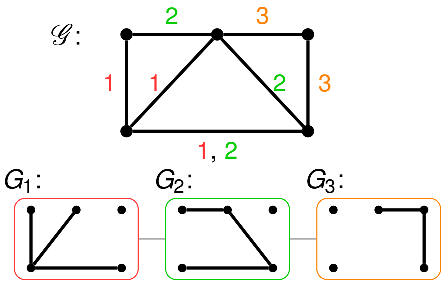

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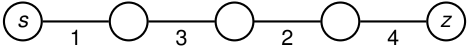

Not a temporal path.

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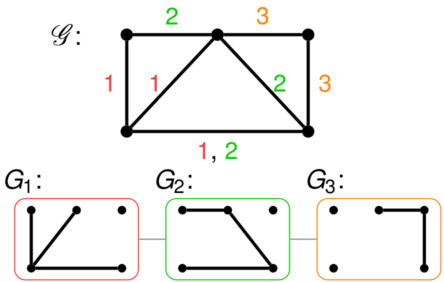

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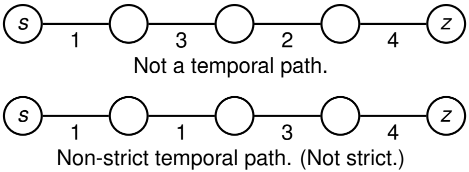

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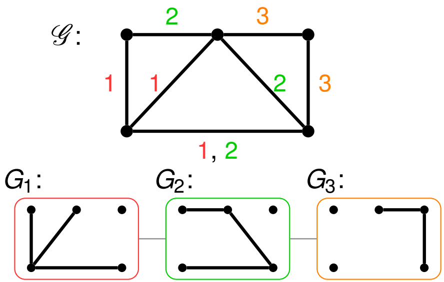

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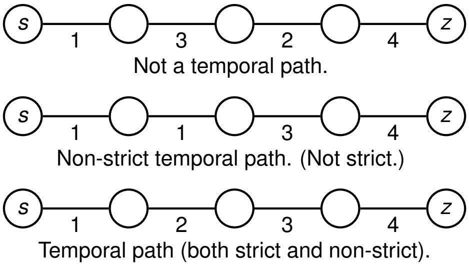

Motivation: Temporal Betweenness

"How important is Berlin main station as a hub for the public transportation network?"

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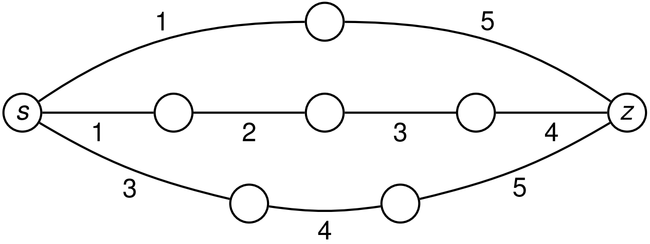
Betweenness of a vertex v in a graph $G = (V, E)$:
"How likely is a shortest (optimal) path to pass through vertex v?"

Optimal Temporal Paths

Which temporal path from s to z is optimal?



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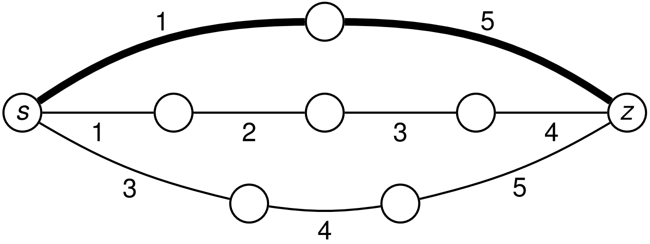

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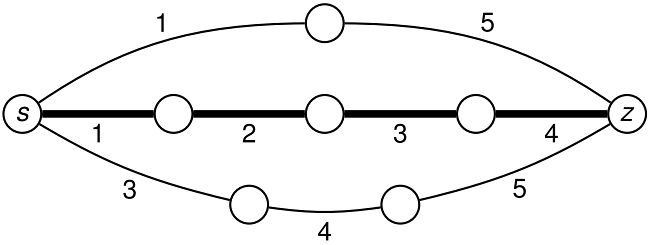

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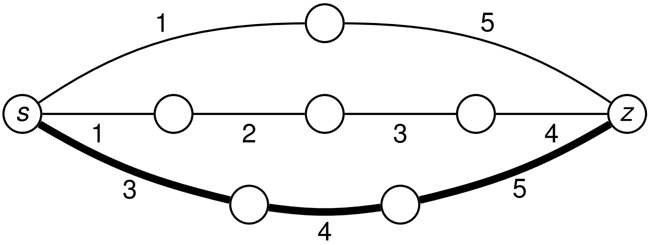

- **Shortest** temporal paths use the minimum number of edges.
- **Foremost** temporal paths have a minimum arrival time.
- **Fastest** temporal paths have a minimum difference between starting and arrival time.

Temporal Betweenness: Definition

Temporal Betweenness Centrality

$$C_B^{(\star)}(v) = \sum_{s \neq v \neq z} \begin{cases} 0 & \nexists \text{ temp.}(s,z)\text{-path} \\ \frac{\sigma_{sz}^{(\star)}(v)}{\sigma_{sz}^{(\star)}} & \text{otherwise} \end{cases}$$

$\sigma_{sz}^{(\star)}$: # $\star$-temp. paths from s to z

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- Computing temporal betweenness essentially equivalent to counting (optimal) temporal paths.
- For many interesting optimality concepts such as "foremost" or "fastest" the corresponding counting problem is **#P-hard**.

Temporal Path Counting I

#Temporal Path

Input: A temporal graph $\mathcal{G} = (V, (E_i)_{i \in [T]})$ and two vertices $s, z \in V$.

Task: Count the temporal (s, z)-paths in $\mathcal{G}$.

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- For this talk: focus on **non-strict** temporal paths.

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#Temporal Path is #P-hard even if $T = 1$.

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Approximate Counting

Warm-Up: Temporal Path Counting on Forests

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#Temporal Path solvable in polynomial time if the underlying graph is a **forest**.

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Question: How can we generalize the forest algorithm?

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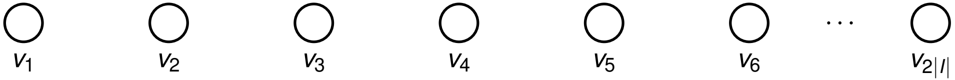
- Model one track with vertices and one with time.

Parameterized Hardness II



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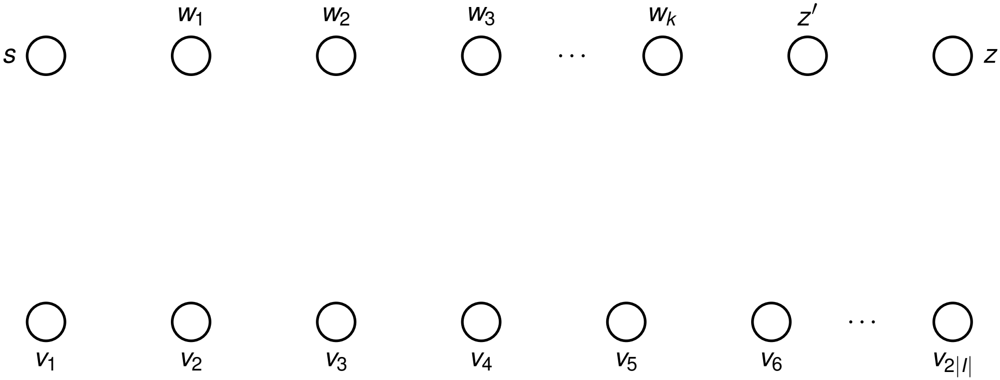

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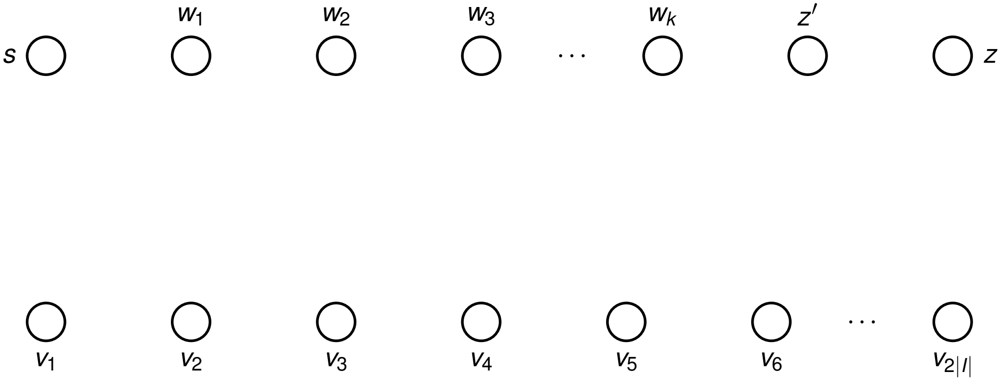

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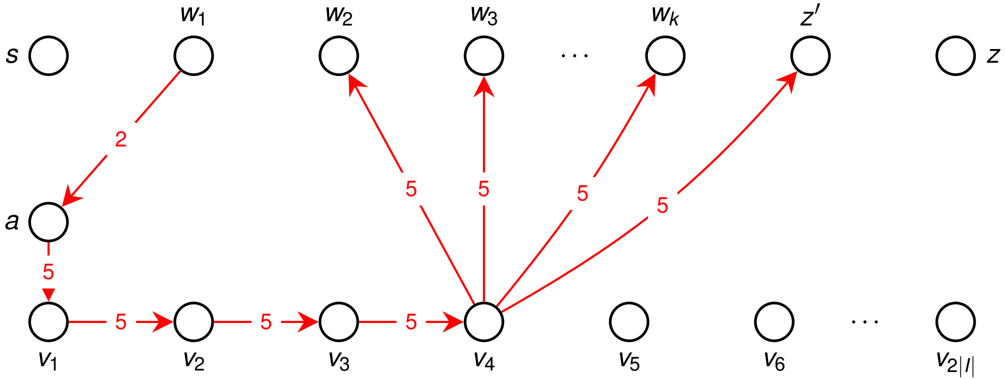

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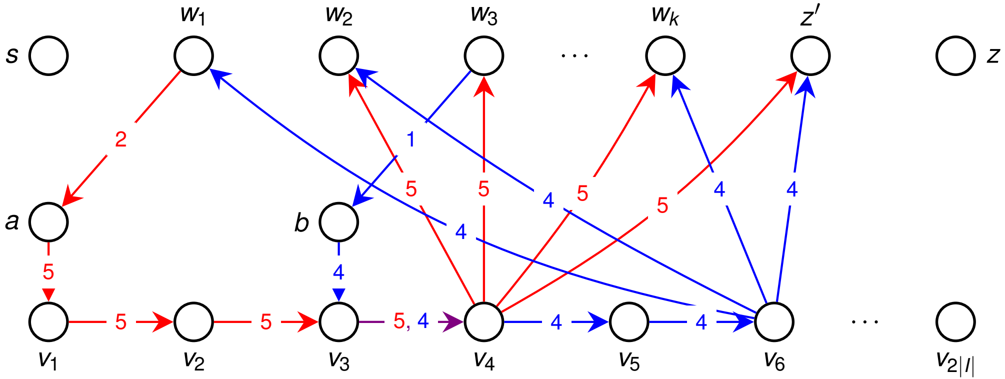

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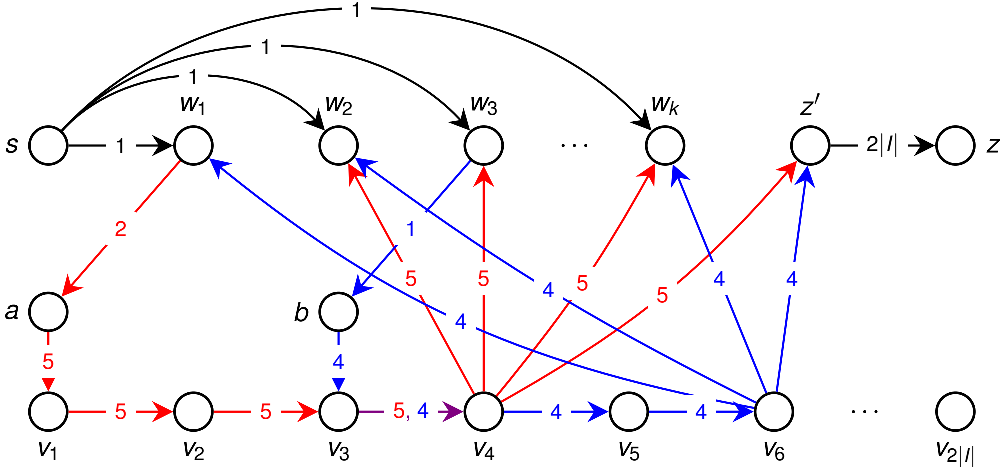

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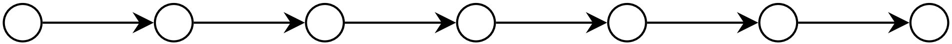

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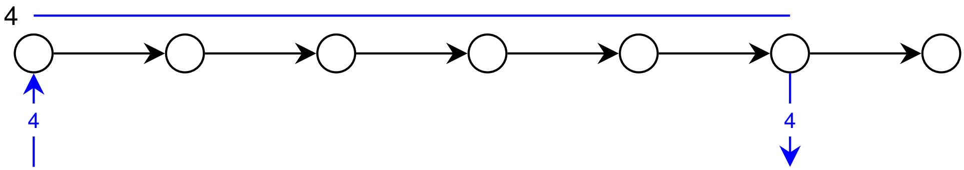

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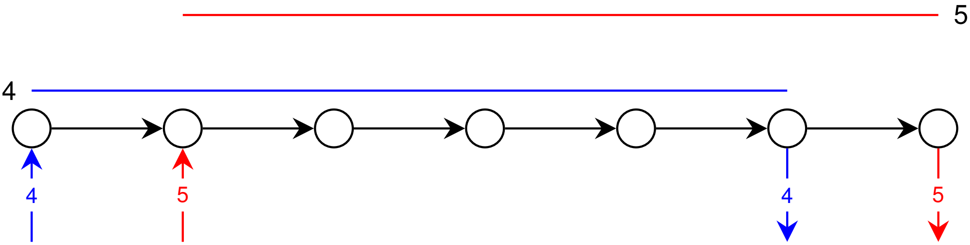

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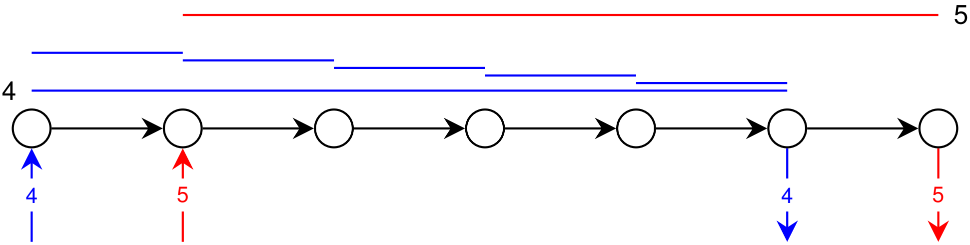

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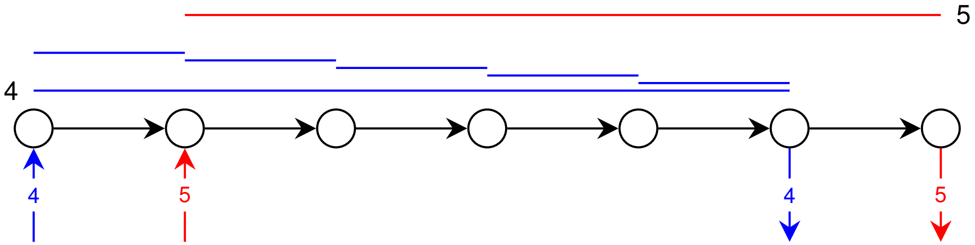

Lemma

The parity of temporal (s, z)-paths equals the parity of **colorful** independent sets.

Generalizations of the Forest Algorithm and Further Results

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Theorem

#Temporal Path is in FPT when parameterized by the **treewidth** of the underlying graph and the **lifetime** T.

Theorem

#Temporal Path is in FPT when parameterized by the **vertex-interval-membership-width**.

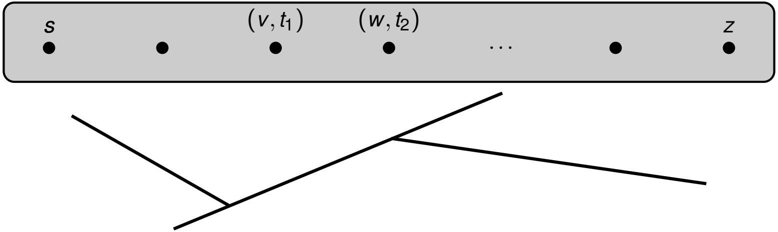
Timed Feedback Vertex Number I

Timed Feedback Vertex Set

Minimum size $X \subseteq V \times [T]$ such that underlying graph of $\mathcal{G} - X$ is a forest.

TFVS:

Forest:



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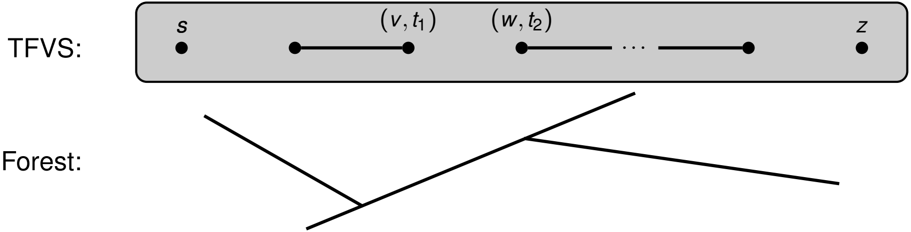

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Timed Feedback Vertex Number I

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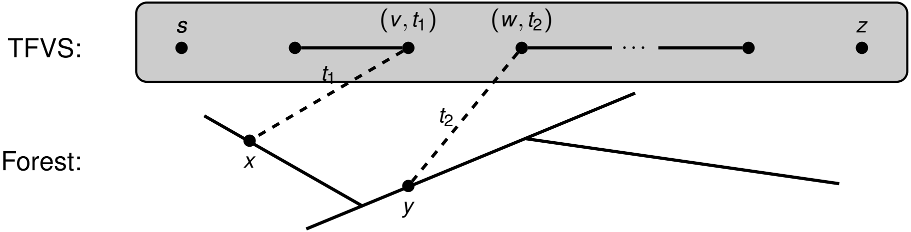

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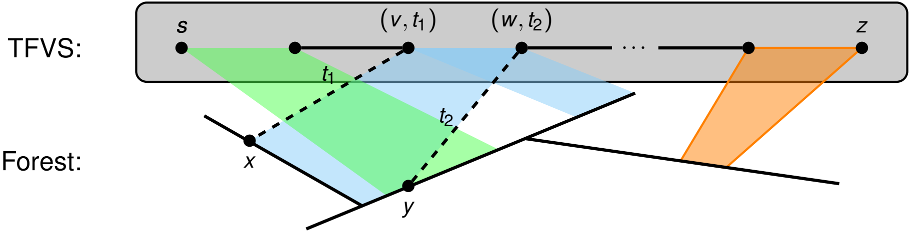

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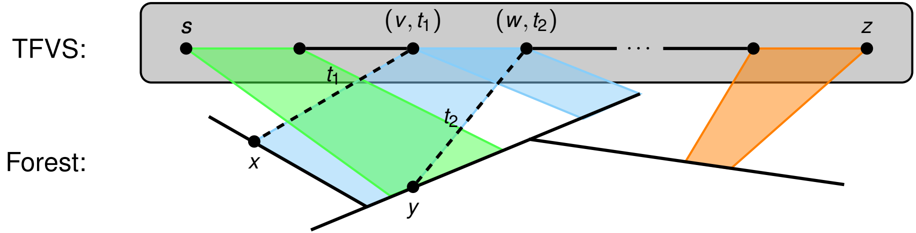

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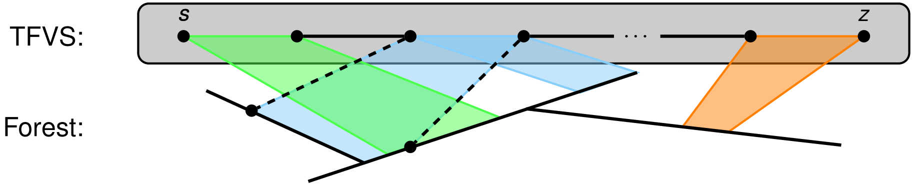
- Use forest algorithm to compute all ways to close "gaps" between TFVS appearances.
- For every valid way to glue together a temporal (s, z)-path, the number of temporal (s, z)-paths visiting the same vertex sequence equals the **product** of the number of ways to close each gap.

Timed Feedback Vertex Number II



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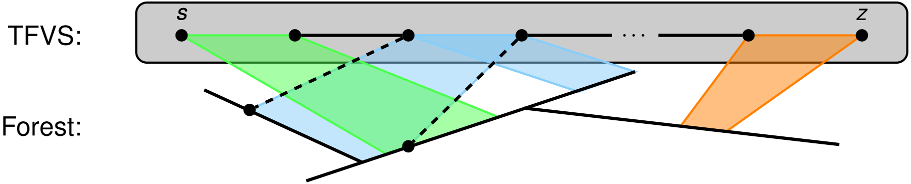

Note: Intersection graphs of paths segments in forests are **chordal**.

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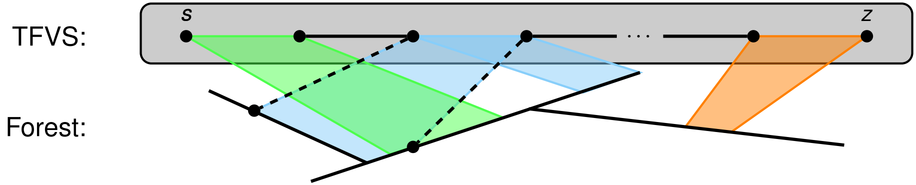

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#Weighted Multicolored Independent Set on Chordal Graphs

Input: A chordal graph $G = (V, E)$, a coloring $c : V \to [k]$, and weights $w : V \to \mathbb{N}$.

Task: Compute $\sum_{X \subseteq V \mid X \text{ is a multicolored independent set in } G} \prod_{v \in X} w(v)$.

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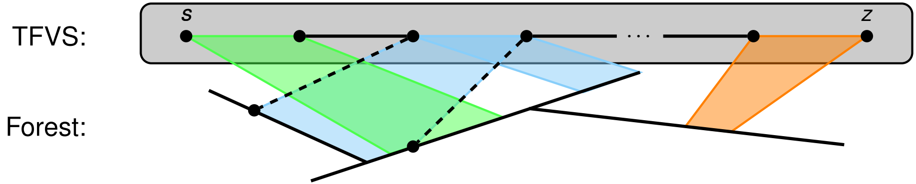

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Idea: Adapt FPT algorithm for **Multicolored Independent Set on Chordal Graphs** parameterized by number of colors by Bentert, van Bevern, and Niedermeier [JOSH 2019].

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Link to arXiv.

Thank you!