Two Influence Maximization Games on Graphs Made Temporal[1]

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ICALP 2021
Workshop on Algorithmic Aspects of Temporal Graphs IV
July 12, 2021

[1] Accepted at IJCAI 2021

Influence Maximization Games

A game-theoretic model for studying

- spread of information/influence,
- opinion diffusion (viral marketing, "word-of-mouth"),
- dissemination processes (disease spreading),
- ...

Model:

- Undirected (static) graph $G = (V, E)$ and $k \geq 2$ players.
- Players choose a subset of V in order to "influence" (color) as many other vertices as possible.
- Payoff of a player is the number of vertices she colored.

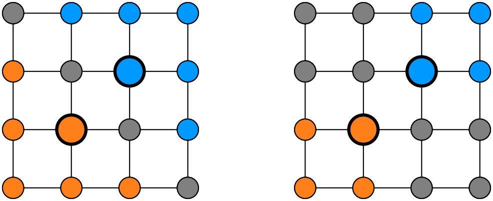
Two Games on Static Graphs

Here: $k = 2$ players selecting one vertex (simultaneously).

Diffusion Game: Vertices are colored by a propagation process:

- An uncolored vertex having only one player's color in its neighborhood is colored with this color.
- A vertex with both colors in its neighborhood is colored "gray" (standoff).

Voronoi Game: A player colors all vertices that are closest to her.



Diffusion   Voronoi

Related Work

Diffusion Games:
Introduced by Alon, Feldman, Procaccia, and Tennenholtz (IPL '10).

Existence of Nash equilibria on various graph classes for different numbers of players well studied.
[IPL '12, IPL '13, AAIM '14, WADS '15, DAM '16, IM '16, SOFSEM '20]

Voronoi Games:
Introduced by Dürr and Thang (ESA '07).

Focus also on existence of Nash equilibria on graph classes.
[MFCS '08, WINE '09, TCS '15, TCS '20]

Contributions

Fact: Real-world (e.g. social) networks change over time.

$\rightsquigarrow$ Study influence maximization games on temporal graphs.

Temporal graph games not much studied in the literature so far.
(Erlebach and Spooner (SOFSEM '20): Cops-and-Robber game on edge-periodic graphs)

Our Contributions:

- Initiate game-theoretic studies on temporal graphs.
- Generalize Diffusion and Voronoi games to temporal setting.
- Analyze existence of Nash equilibria in temporal paths/cycles.
- Observe complex and quite different behaviour of the two temporal games (in contrast to the static case).

Temporal Graph Games

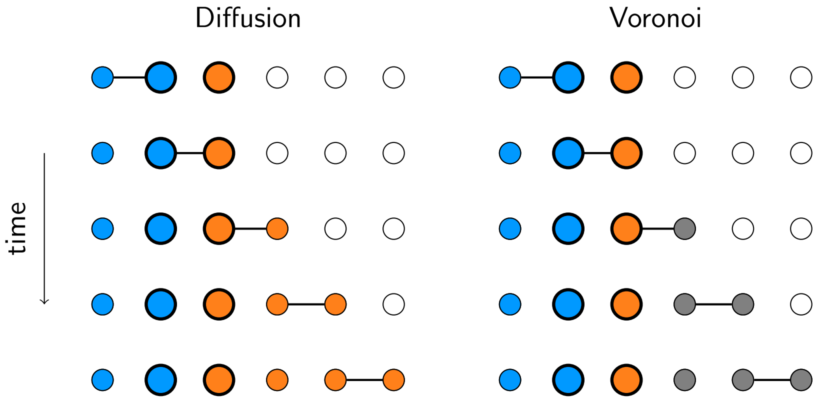
- $\mathcal{G} = (V, E_1, \ldots, E_\tau)$: temporal graph
- E_t: set of edges present in step t

Temporal Diffusion Game: Analogous to static case using edges in E_t in step t of the propagation process.
Propagation continues on $G_\tau = (V, E_\tau)$ until finished.

Temporal Voronoi Game: A player colors all vertices which she reaches earlier (arrival time of a strict temporal walk).
Last layer G_τ may be repeated arbitrarily often (allowing arrival times $> \tau$).

Note: If $E_1 = E_2 = \cdots = E_\tau$, then temporal diffusion and Voronoi games are equivalent to the static variants.

Example



Note: Static diffusion and Voronoi games are identical on paths (and cycles) and always have a Nash equilibrium.

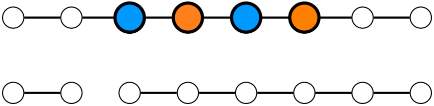
Overview

Our results on guaranteed existence (✓) of a Nash equilibrium:

	Growing ($E_t \subseteq E_{t+1}$)	Superset ($E_t \subseteq E_\tau$)	Shrinking ($E_t \supseteq E_{t+1}$)
Diffusion			
Temporal Paths	✓ $O(n)$	✓ $O(n)$	✗
Temporal Cycles	✓ $O(n\tau)$	✗	✗
Voronoi			
Temporal Paths	✓ $O(n^2)$	✗	✗
Temporal Cycles	?	✗	✗

Temporal path/cycle = underlying graph is a path/cycle

A Shrinking Temporal Path



No Nash equilibrium in diffusion or Voronoi game.

Lemma

Temporal diffusion and Voronoi games are equivalent on shrinking temporal paths and cycles.

Theorem

Temporal diffusion and Voronoi games are not guaranteed to have a Nash equilibrium on shrinking temporal paths or cycles.

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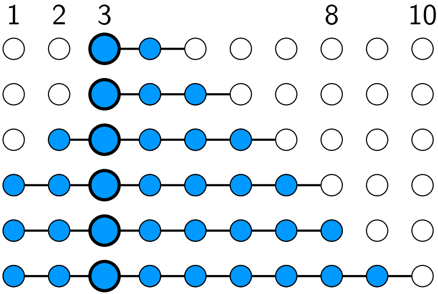
Voronoi Game on Growing Temporal Paths

Theorem

Let $\mathcal{P} = ([n], E_1, \ldots, E_\tau)$ be a growing temporal path. Then, there exists a Nash equilibrium in the temporal Voronoi game on $\mathcal{P}$.

Proof based on analyzing best responses using "boundaries":

A vertex $u \leq v$ is a **left boundary** *of v if v reaches u before the edge $\{u-1, u\}$ appears (right boundaries analogous).*

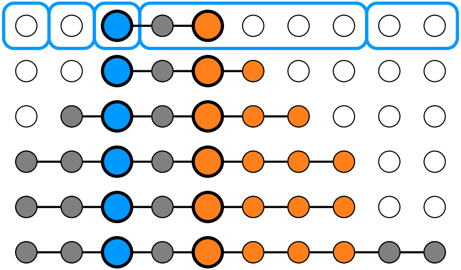


Boundaries of 3: 1, 2, 3, 8, 10

Best Response

Observations:

- Boundaries partition vertices into intervals.
- A player can only color vertices within a boundary interval of the other player.
- Best response: Choose a largest boundary interval I of the other player and choose the vertex in I that is closest to the other player.



Lemma *Every best response sequence contains a Nash equilibrium after at most n steps.*

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Conclusion

Temporal diffusion and Voronoi games are rich of complex behaviour even on temporal paths and cycles.

Forbidding edges to disappear helps to guarantee Nash equilibria.

Future Work:

- Other temporal graph classes
- Model variations (more players, limited time horizon, different payoff)
- Other temporal distances for Voronoi games (non-strict, fastest, shortest)
- Playing other games on temporal graphs

Thank you!