Finding Temporal Paths under Waiting Time Constraints

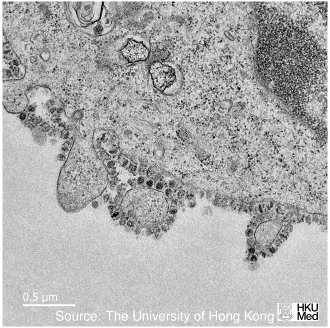
Philipp Zschoche

TU Berlin

July 7 2020, Algorithmic Aspects of Temporal Graphs III

Based on joint work with Arnaud Casteigts, Anne-Sophie Himmel, and Hendrik Molter.

Motivational Example: Disease Control



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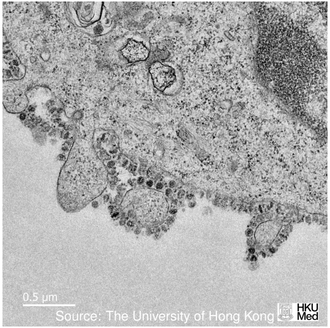

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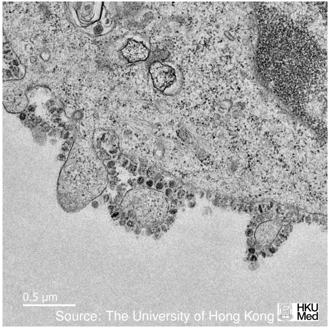

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(2) Isolate all potentially infected persons (by known cases).

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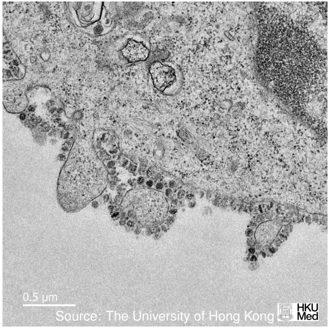

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At time of identification: a person may started **long infection chains**.

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Can s have (indirectly) infected z?

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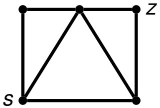

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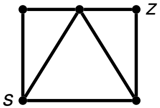

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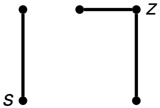

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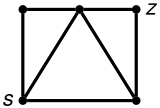

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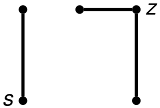

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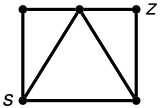

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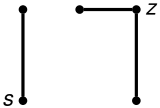

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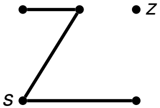

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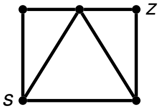

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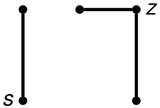

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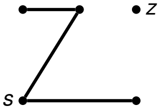

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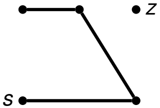

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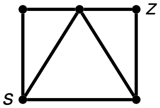

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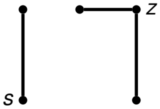

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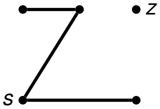

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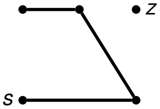

$\Rightarrow$ Time information is crucial for infection transmission routes.

Temporal Graphs – Formal Definition

A **temporal graph** $\mathcal{G} = (V, (E_i)_{i \in [\tau]})$ is defined as vertex set V with a list of edge sets $E_1, \dots, E_\tau$ over V, where τ is the lifetime of $\mathcal{G}$.

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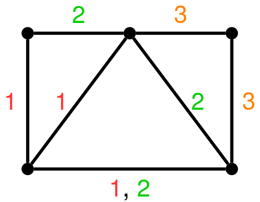

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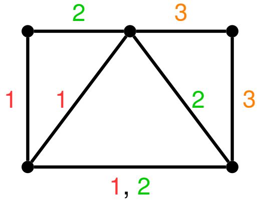
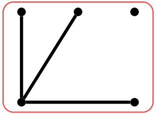


G_1:



layers

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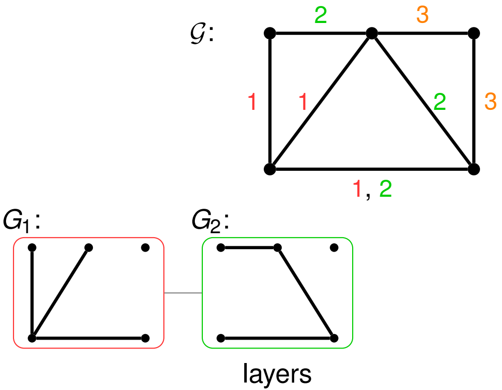

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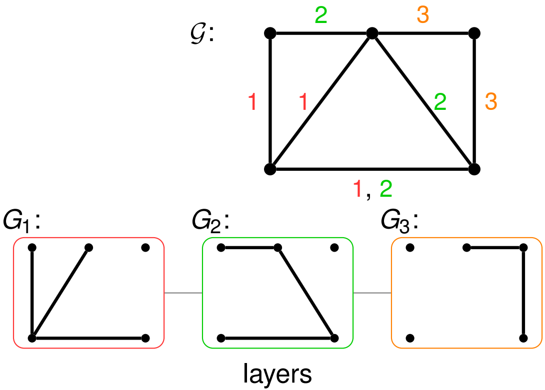

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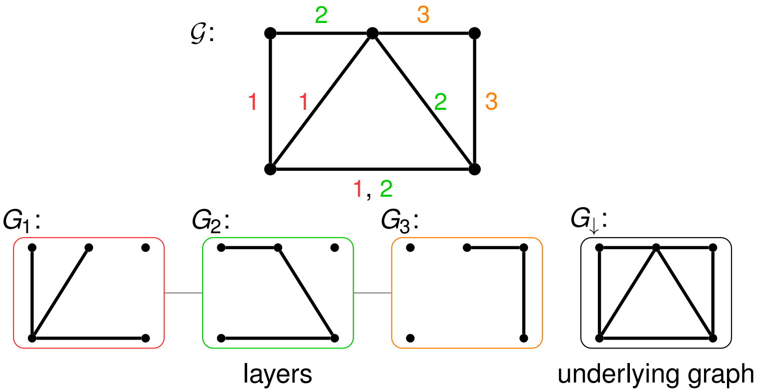
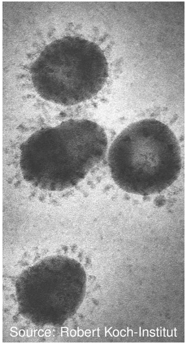


Motivation: Disease Spreading



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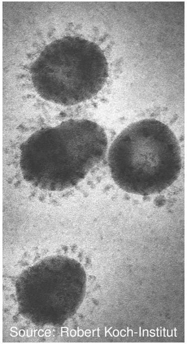
Source: Robert Koch-Institut

Susceptible

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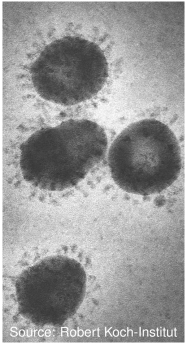

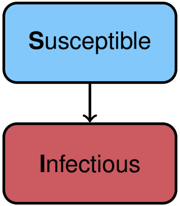

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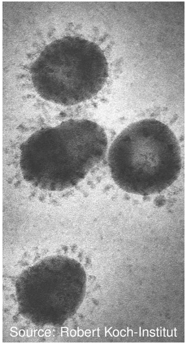

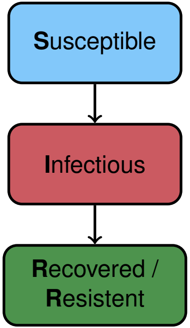

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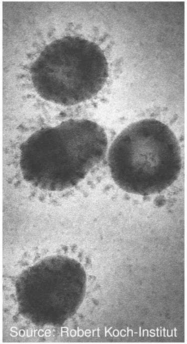

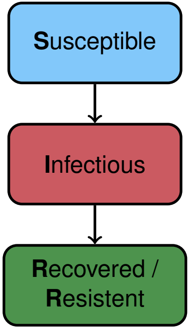

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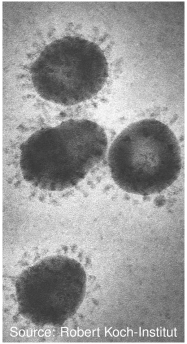

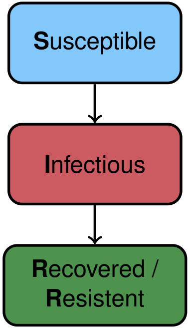

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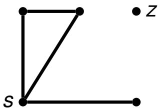

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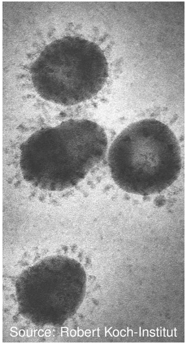

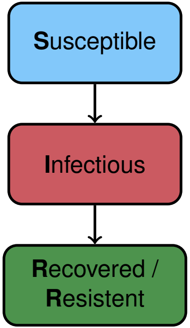

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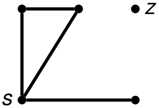

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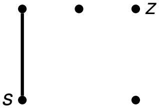

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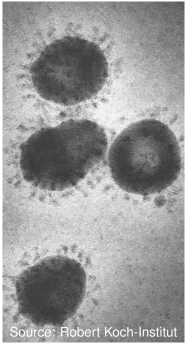

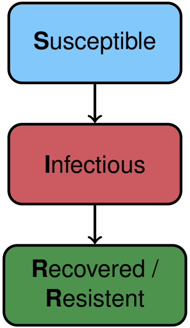

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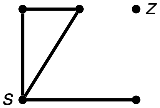

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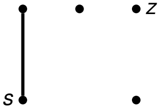

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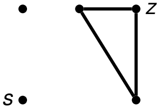

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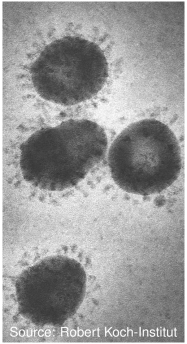
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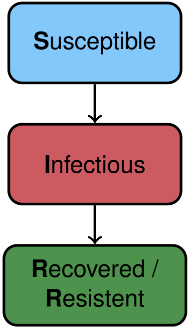

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- Disease spreads along **paths** with **bounded waiting times**.

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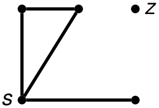

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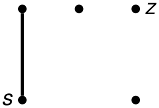

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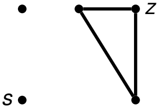

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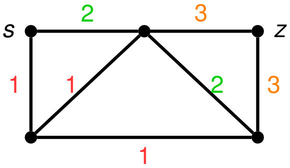

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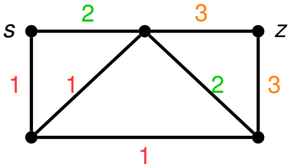

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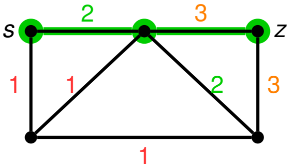

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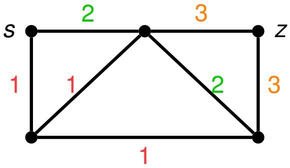

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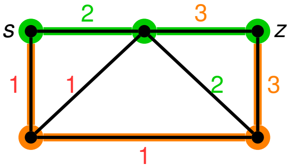

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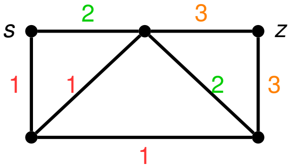

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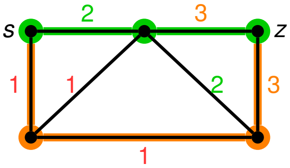

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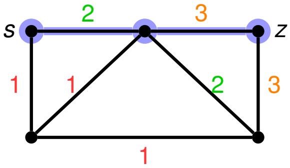

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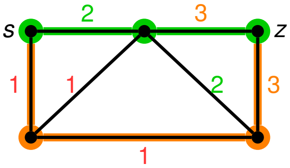

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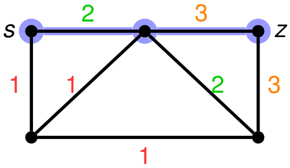

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Restless Paths – Formal Definition

A **Δ-restless temporal** (s, z)**-path** is a list of edges labeled with **non-decreasing** time steps that

- the edges form an (s, z)-path in the underlying graph and
- consecutive time steps differ by at most Δ.

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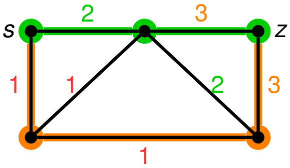

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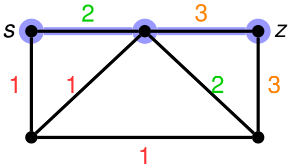

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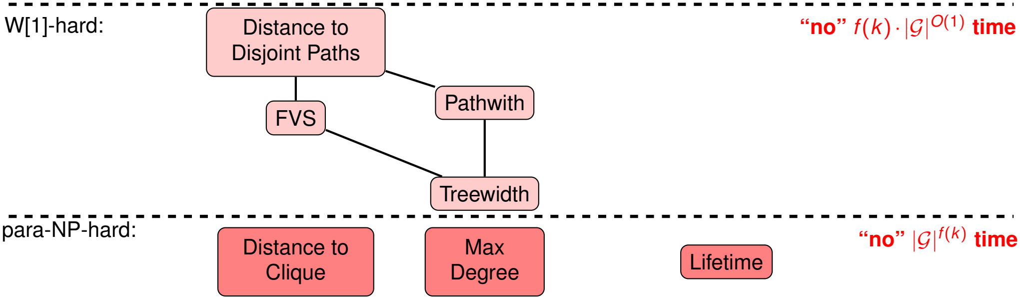

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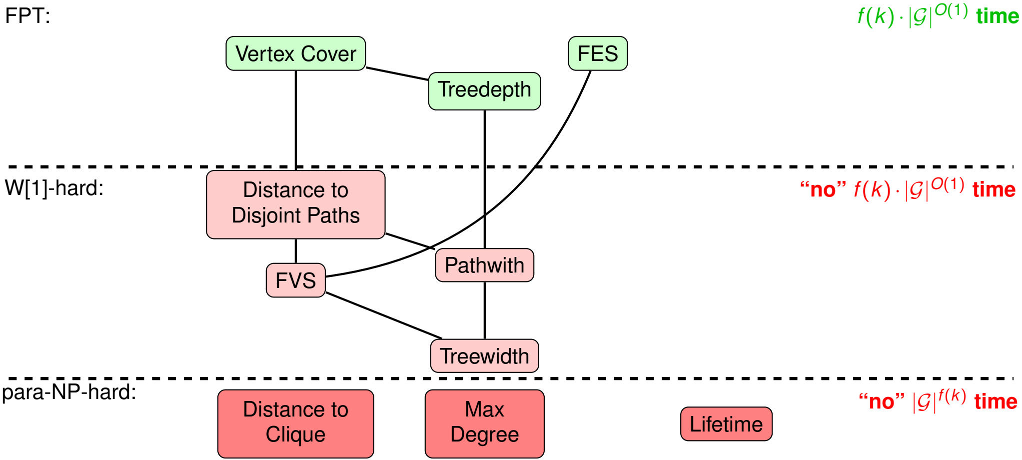

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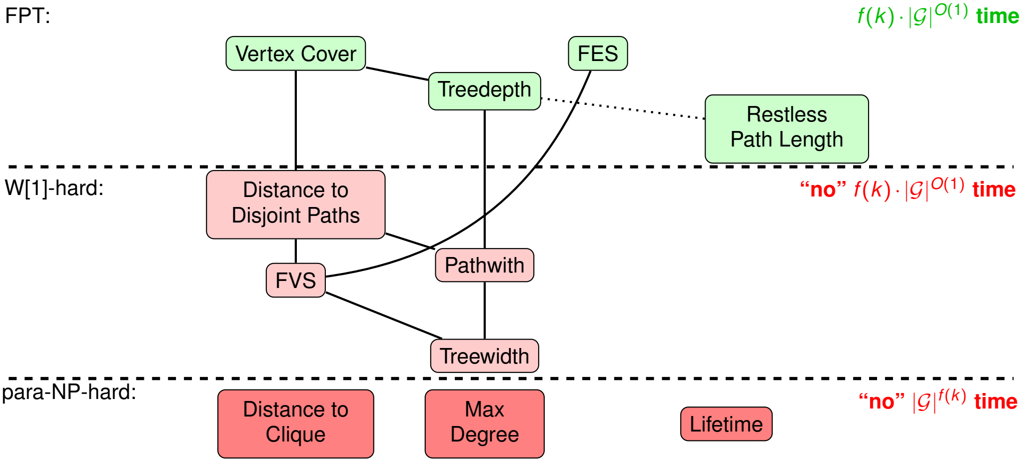

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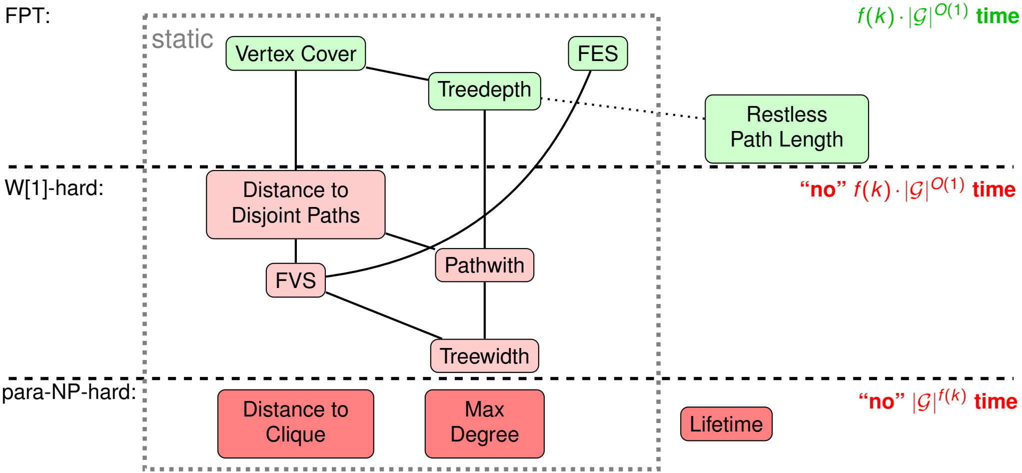

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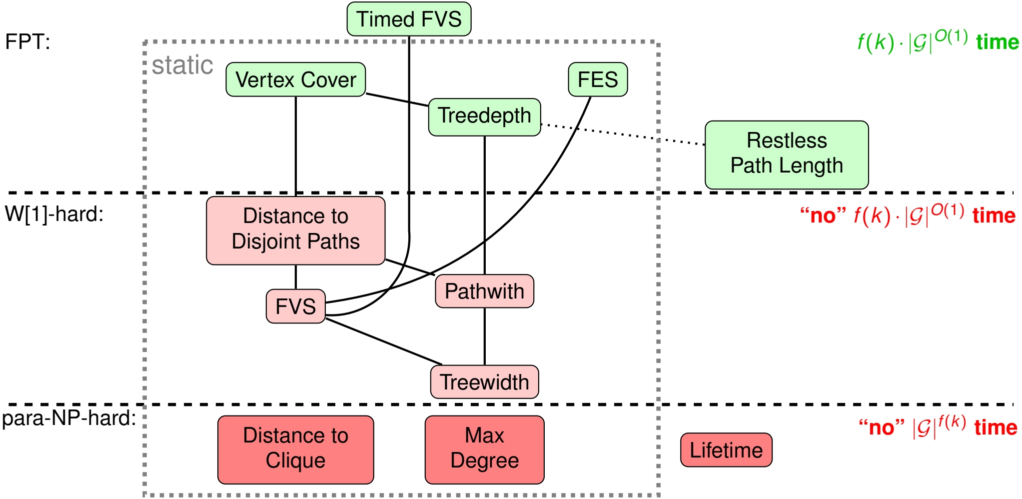

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Today: Lifting **Feedback Vertex Set** for path-related temporal problems.

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Underlying FVS:
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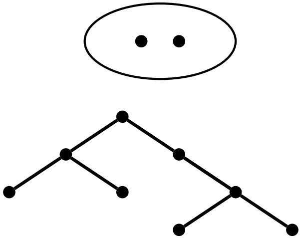

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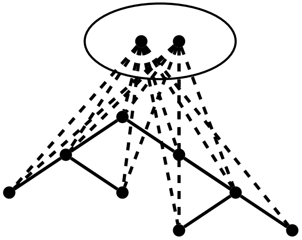

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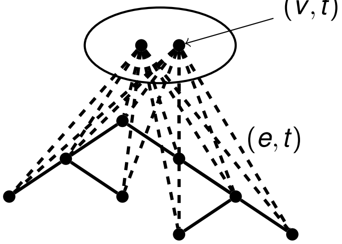

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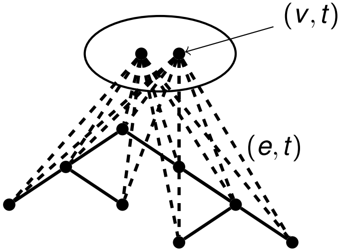

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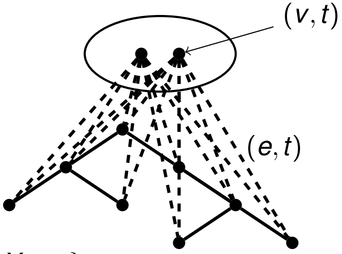

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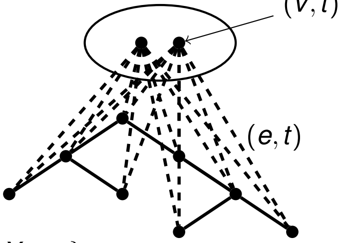

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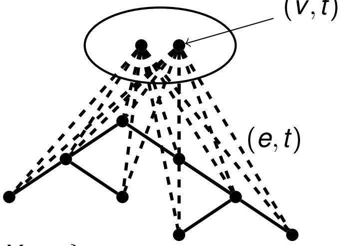

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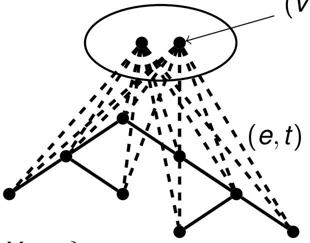

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algorithmically useful: **NP-hard**, but **FPT**. $O^*(4^x)$ time

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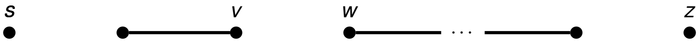

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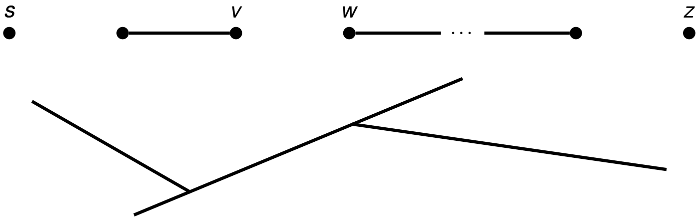

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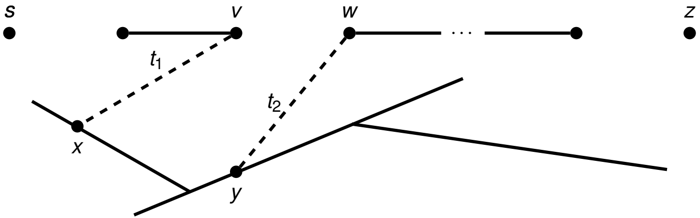

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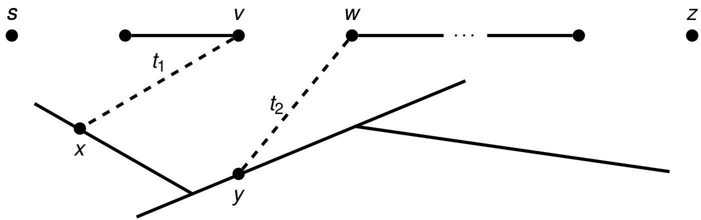

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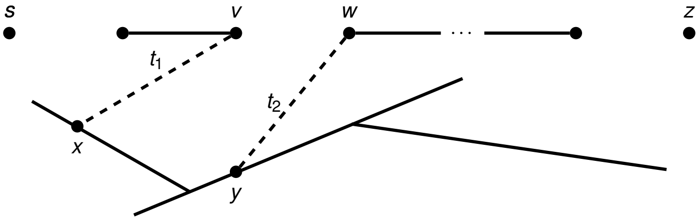

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Resolve Unknown Parts

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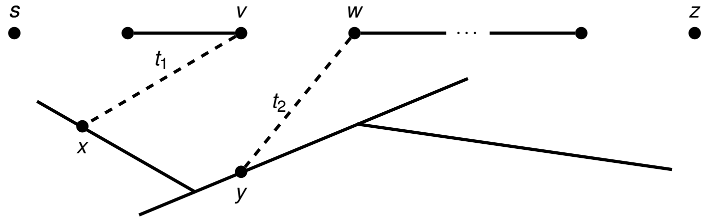

For time edge $e_1 = (\{v, x\}, t_1)$ and $e_2 = (\{w, y\}, t_2)$:

- Is departure time t_1 and arrival time t_2 **"valid"** (4 cases again)?
- Is there a path P from x to y in $T_\downarrow$?
- Is there a Δ-restless path v to w in $\mathcal{G}[P]$ plus e_1, e_2?

Lemma

Restless Path linear time solvable on path.

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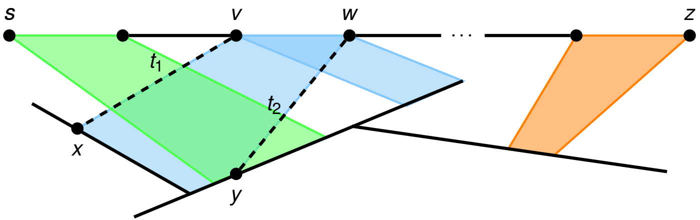

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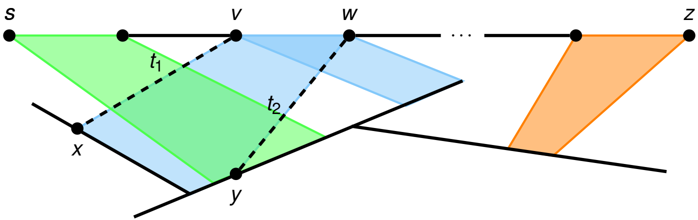

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Intersection graphs of paths in trees are **chordal**.

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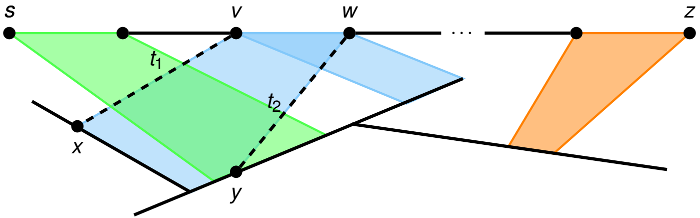

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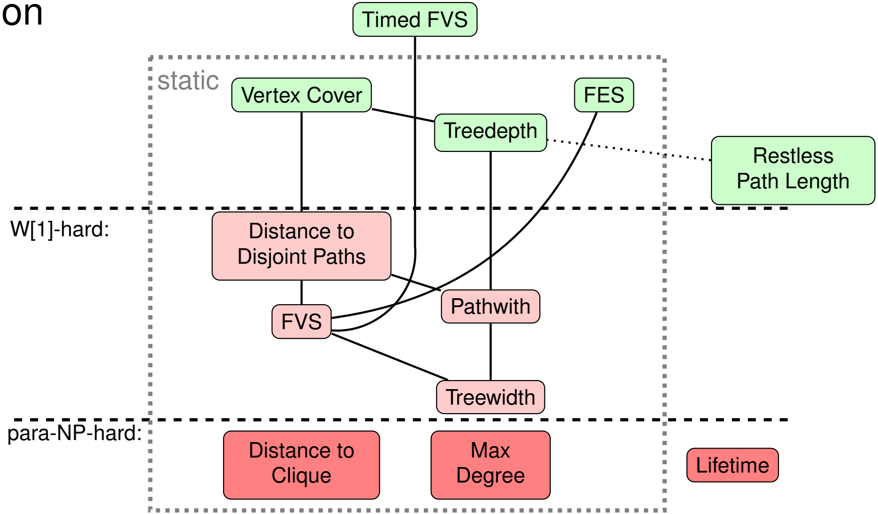
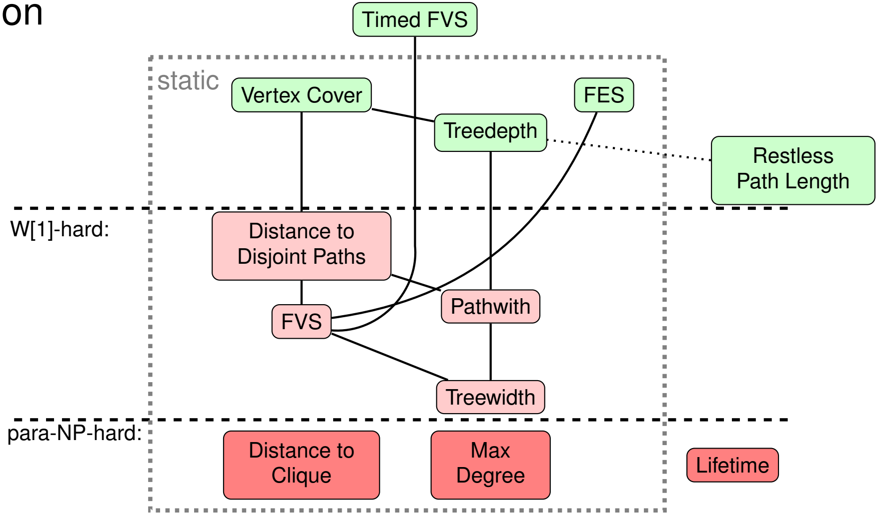
Lemma (Bentert et al., J. Scheduling 19)

Multicolored Ind. Set on Chordal Graphs is **FPT** with # of colors.

Conclusion

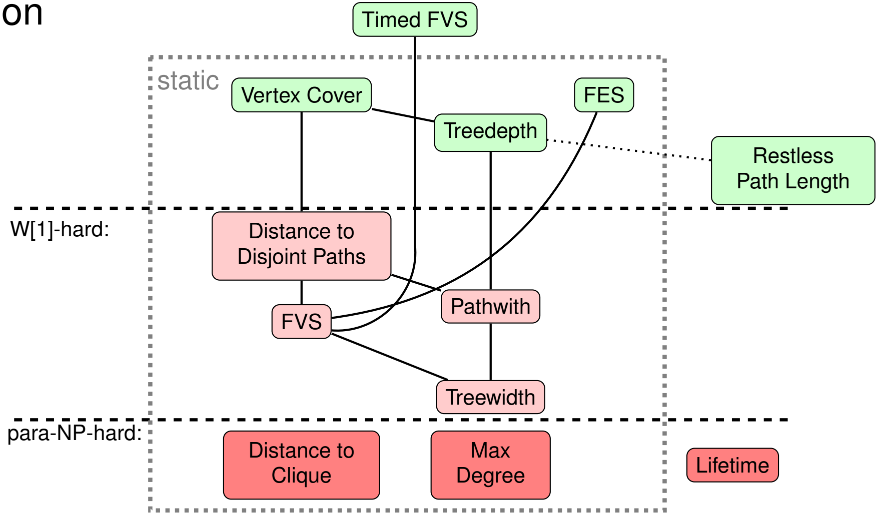

Timed FVS
static
Vertex Cover
FES
Treedepth
Restless
Path Length
W[1]-hard:
Distance to
Disjoint Paths
Pathwith
FVS
Treewidth
para-NP-hard:
Distance to
Clique
Max
Degree
Lifetime

Conclusion



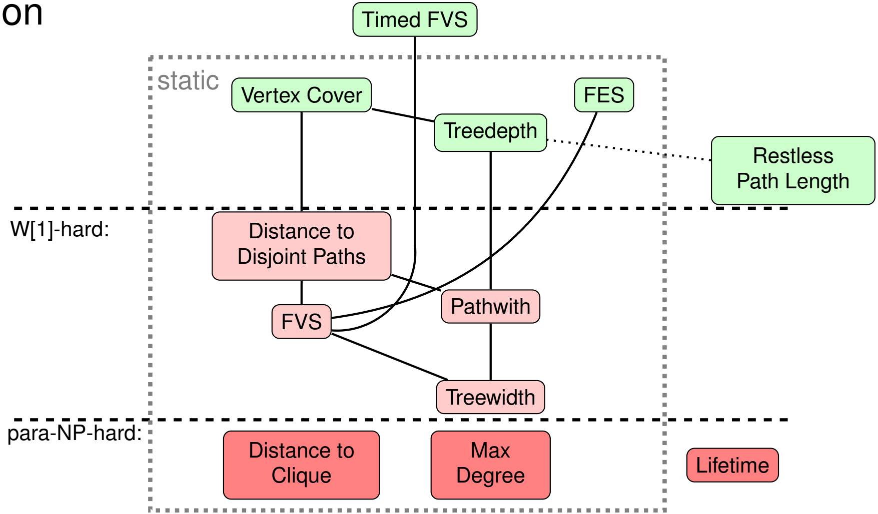
Future: temporal parameters!

Conclusion



Future: temporal parameters!
How to design?

Conclusion



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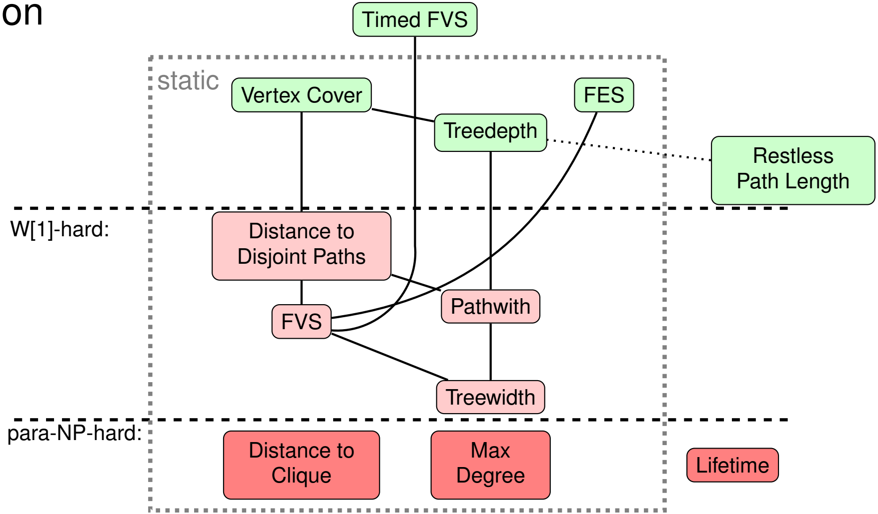

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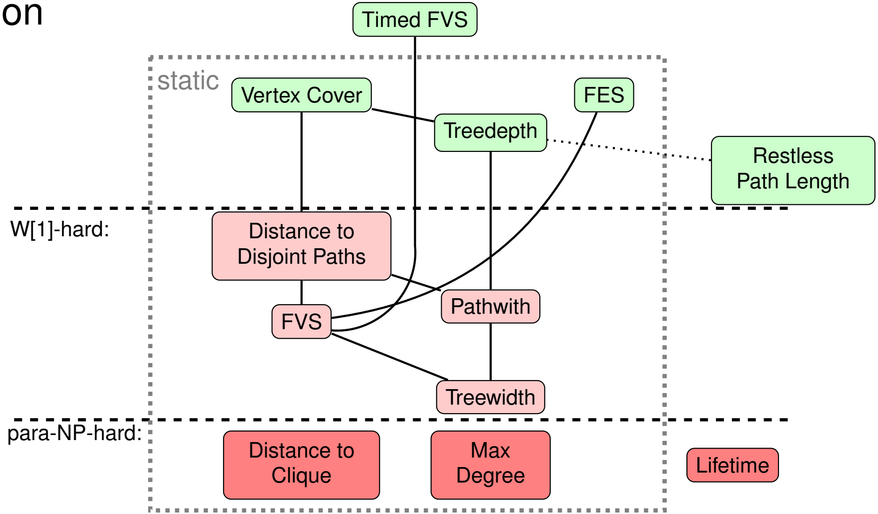

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Thank you!