Comparing Temporal Graphs

with Time Warping

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Joint work with
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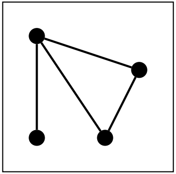
Temporal Graphs



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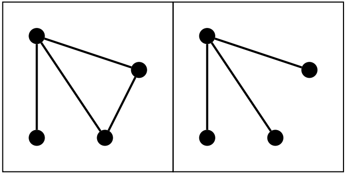

$t = 1$ $t = 2$

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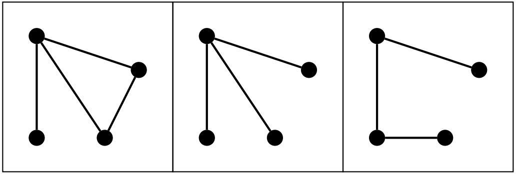

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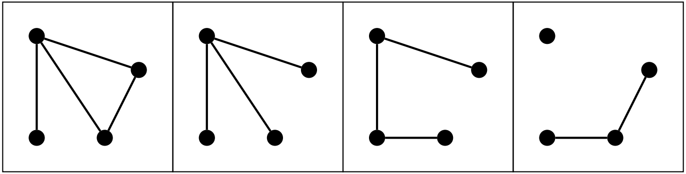

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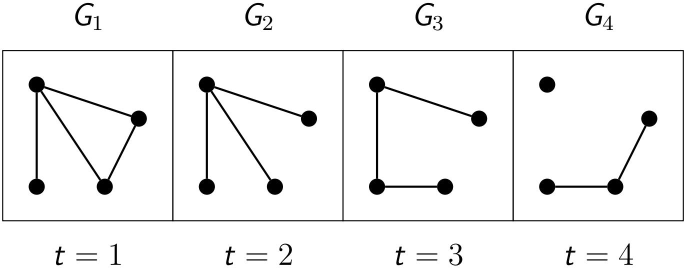

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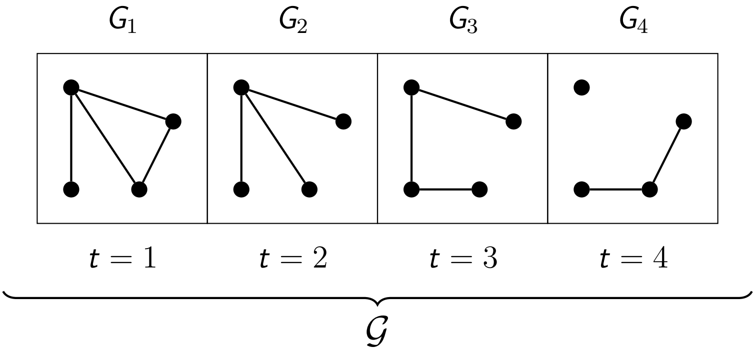

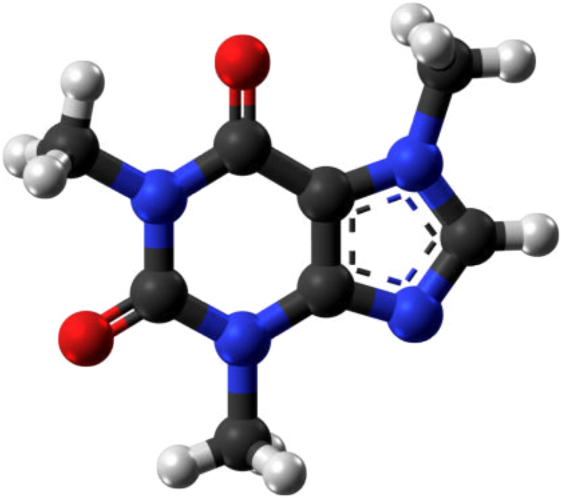

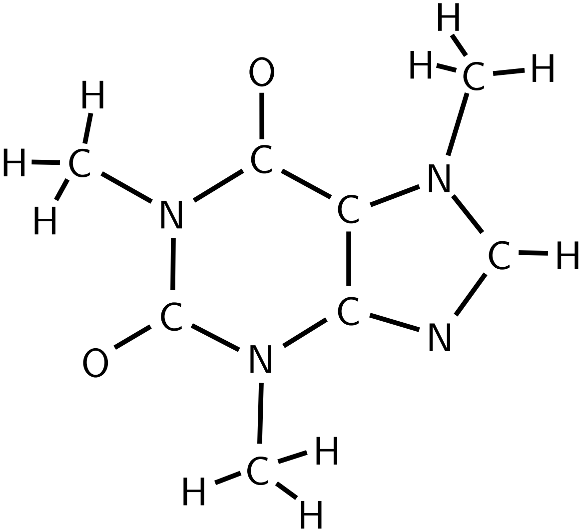
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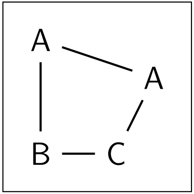
$$\overset{?}{\cong}$$

Main Question

How can we measure the similarity / distance between two temporal graphs $\mathcal{G}$, $\mathcal{H}$?

Graph distance using vertex signatures

G



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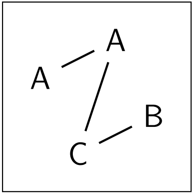

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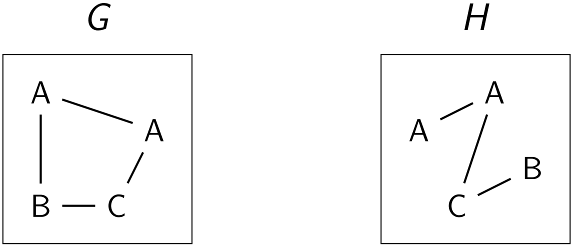

Example vertex signatures:

$\sigma(v) = ($label of v, labels of v's neighbors$)$

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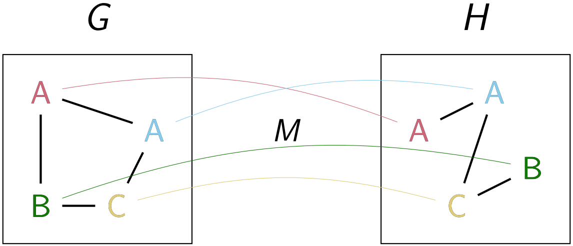

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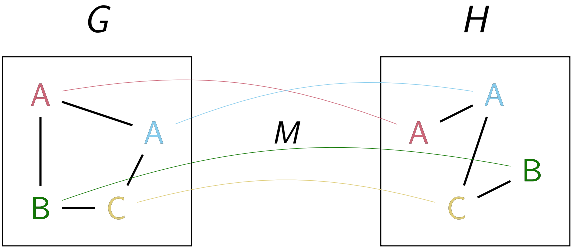

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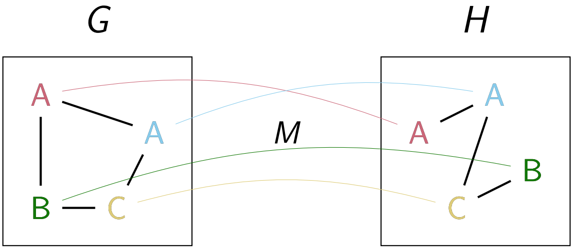

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all maximal matchings between the two vertex sets

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- Computation in cubic time using Jonker-Volgenant (or Hungarian).

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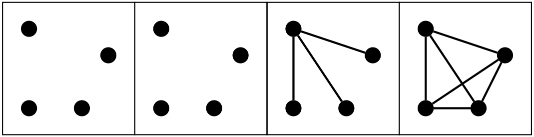
$\mathcal{G}$ 

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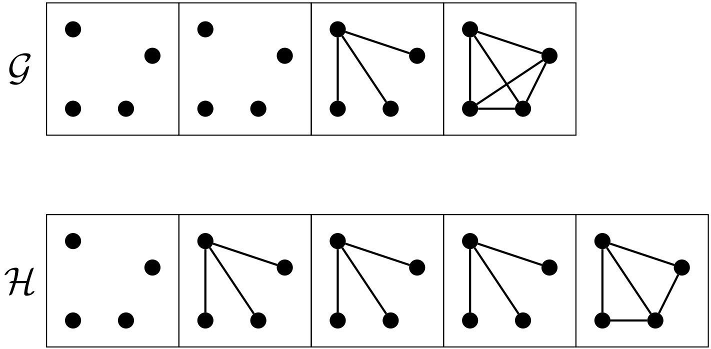

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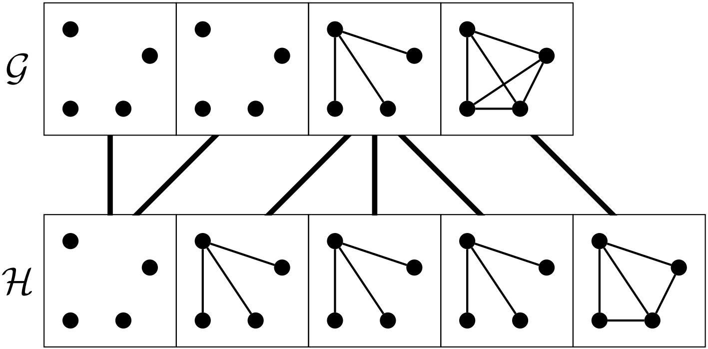

Solution: **Time warping** — assign each layer to the other one it resembles most (no crossings allowed!).

dtgw-distance

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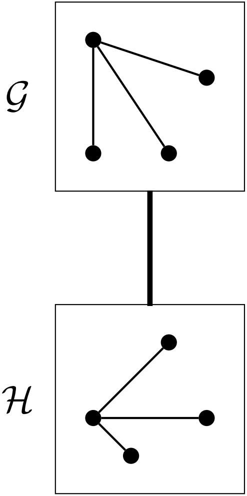
Good news: Time warping can be solved by a dynamic program in quadratic time ...

Bad news: ... if all pairwise distances are known in advance.

Pairwise distances



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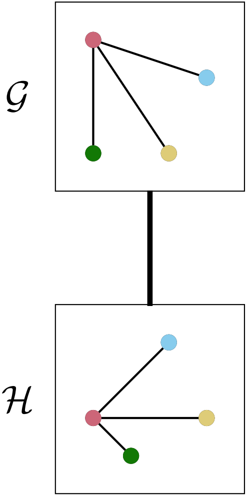
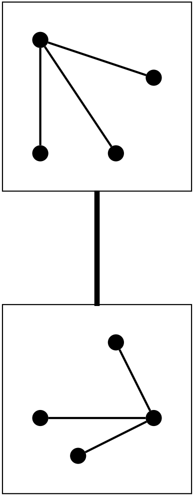


Pairwise distances

$\mathcal{G}$



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Pairwise distances

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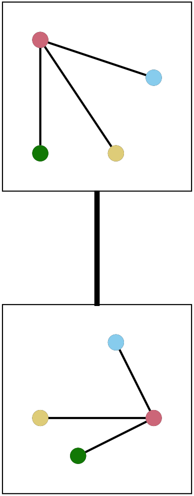

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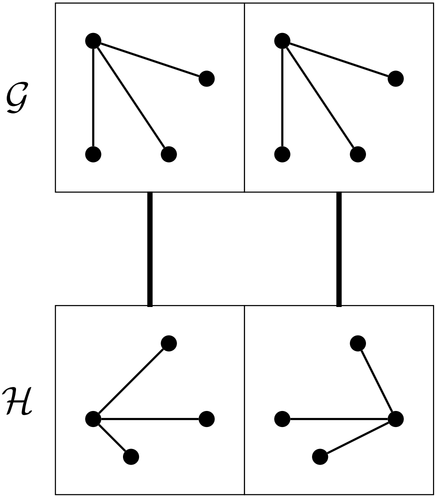

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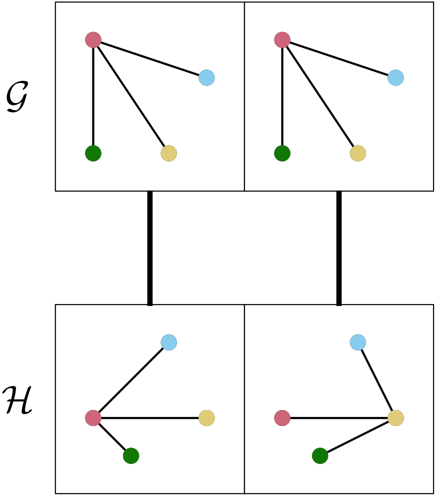

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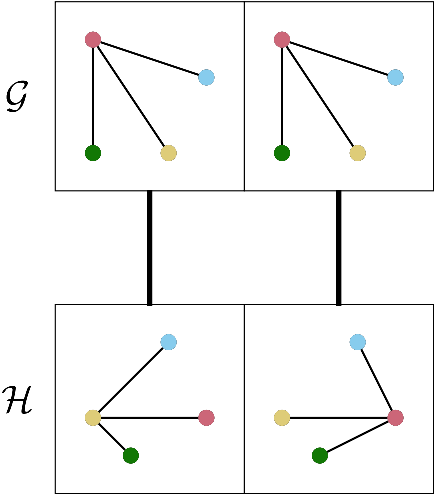

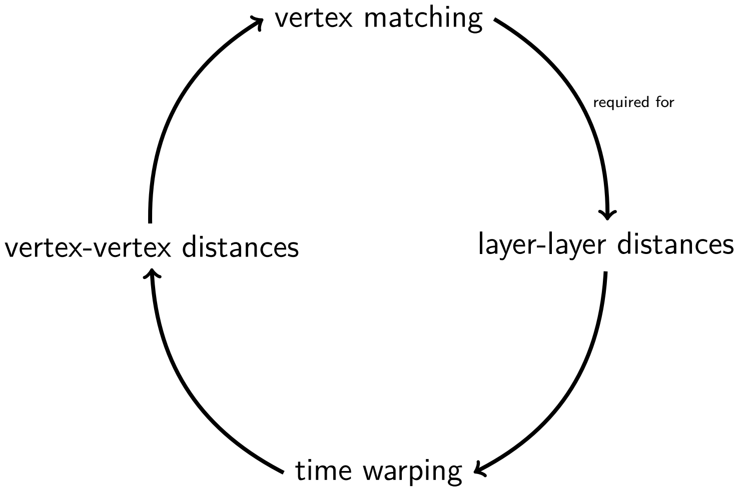
vertex matching
required for
layer-layer distances
time warping
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But ...

- ... you can check in polynomial time whether $\mathrm{dtgw\text{-}dist}(\mathcal{G}, \mathcal{H}) = 0$.

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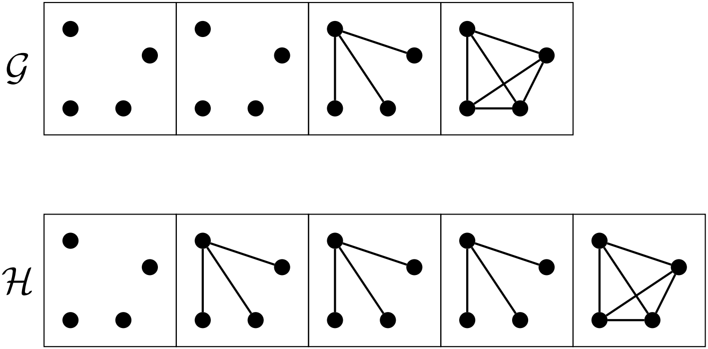
Testing for $\text{dtgw-dist}(\mathcal{G}, \mathcal{H}) = 0$



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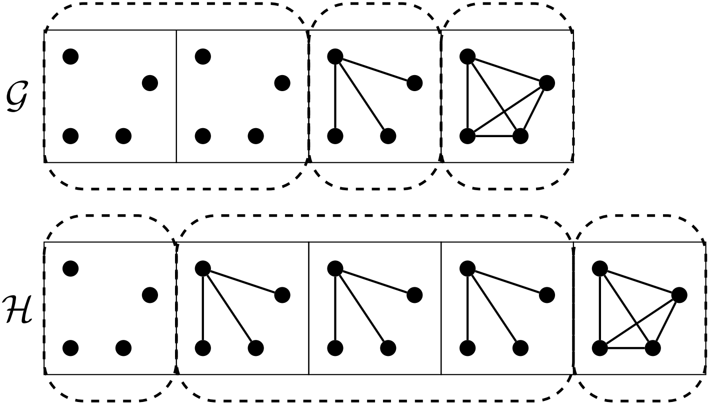

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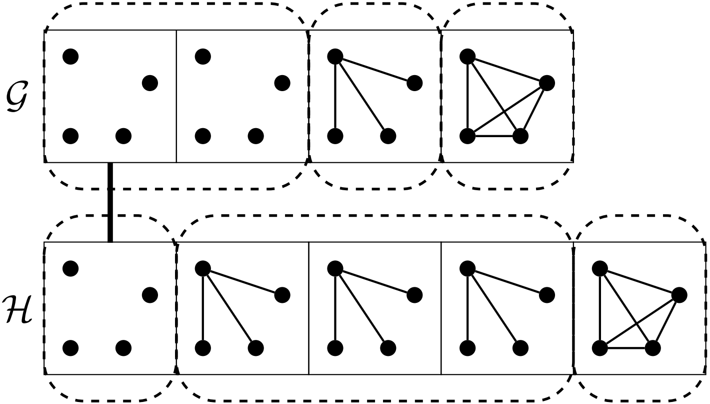

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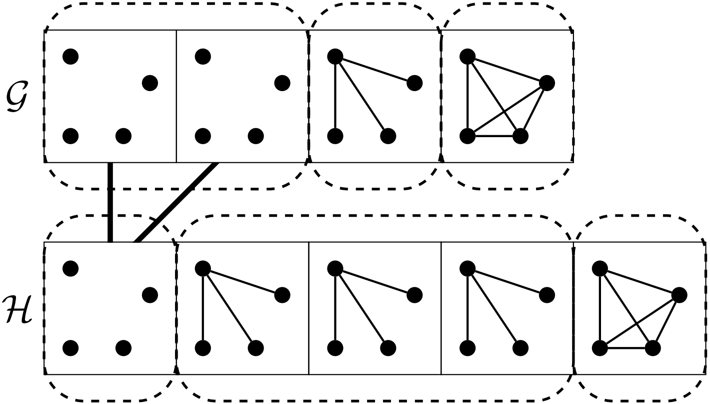

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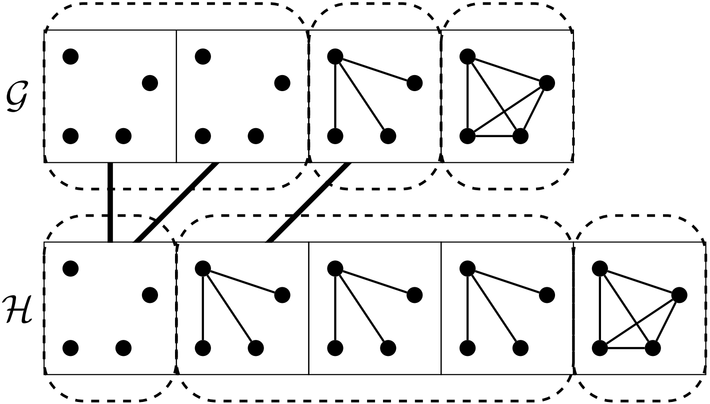

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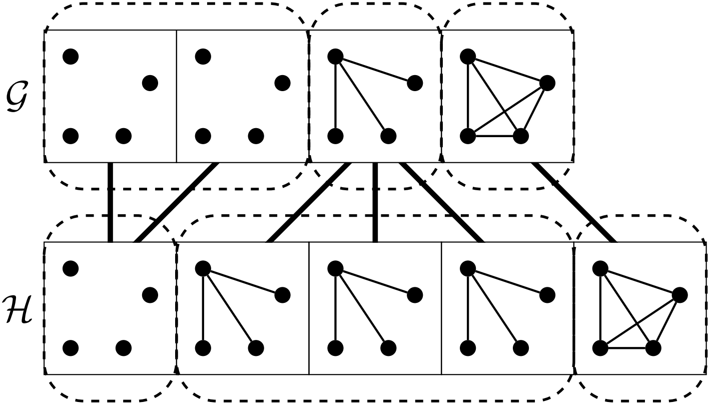

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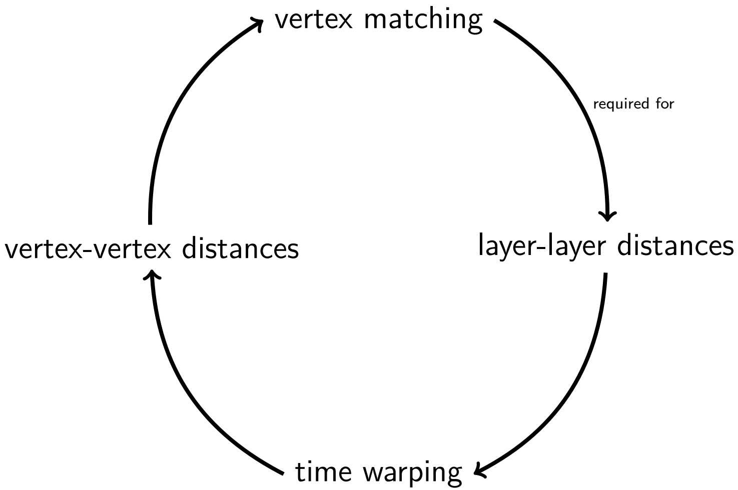
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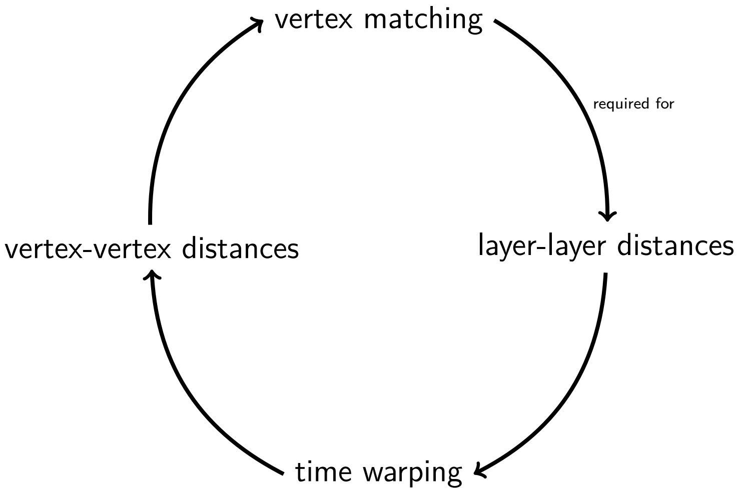

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- Seems to produce good results.

Experiment: Noise

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[2] sociopatterns.org

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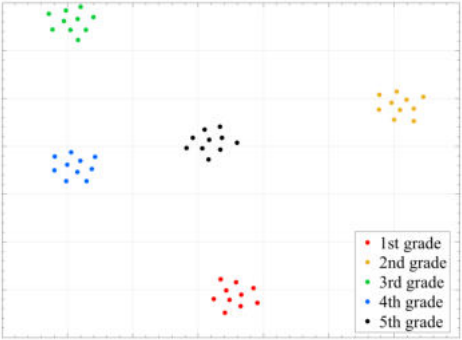

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- Can you find approximation algorithms with guaranteed approximation quality?
- Which vertex signatures work best in different settings?